EESC 9945

Geodesy with the Global Positioning System

Class 12: *GPS Errors*

Systematic and Random Errors in GPS

- Model for the observables

  $y = f(x) + \epsilon \qquad \Delta y = y - f(x_\circ) = A(x - x_\circ) = A\Delta x$

- f is a known mathematic relationship between parameters x (values unknown) and observations y

- The values ϵ are "observational error:"
  - Random, values unknown
  - Zero-mean
  - Gaussian

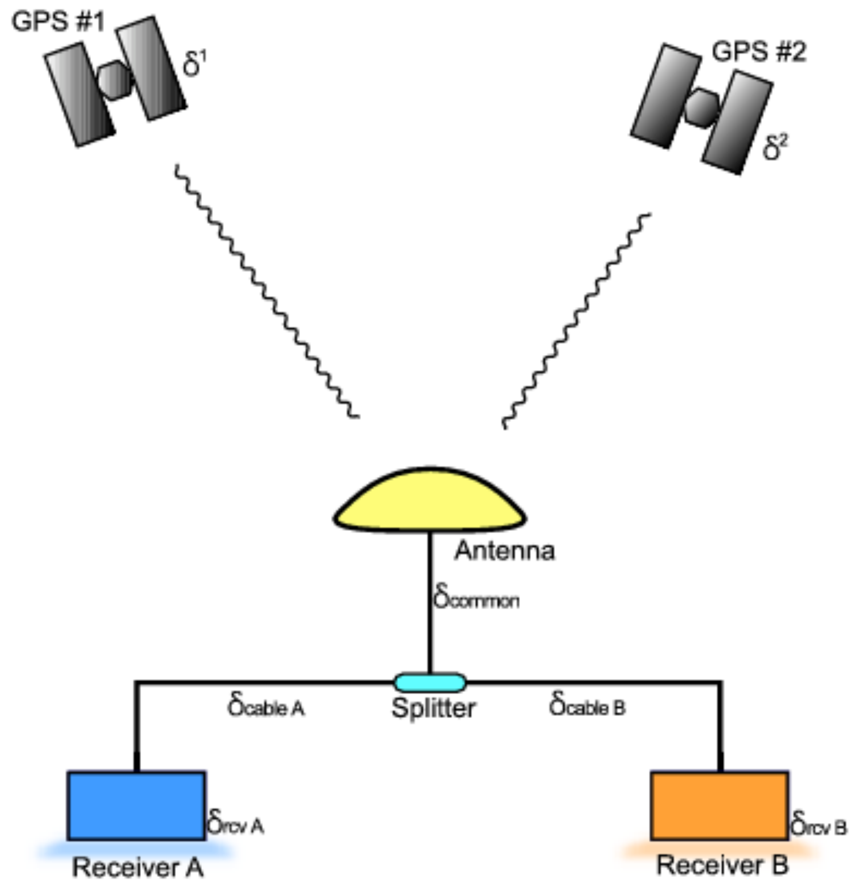
Systematic and Random Errors in GPS

- For GPS, the observational errors ϵ represent an accumulation of errors introduced into the system, including:
  - Inherent noise in signal
  - Ambient RFI, including atmospheric blackbody radiation and cosmic background
  - Instrumental noise in signal detection & phase measurement

Measuring GPS receiver system noise

- The challenge in estimating the observational noise standard deviation σ_ϵ is that there are so many other unknowns to be estimated
  - Clocks
  - Atmosphere
  - Position
  - Orbits
- Most of these unknowns can be eliminated through **zero-baseline measurements**

Zero-Baseline Measurements



http://http://xenon.colorado.edu

Zero-Baseline Receiver Measurements

- Geometric term of the single difference phase ($\Delta\phi(t) = \phi_A(t) - \phi_B(t)$) for small baselines is

$$\Delta\rho(t) \simeq \vec{b} \cdot \hat{e}(t)$$

  where $\hat{e}$ is the unit topocentric vector from the site to the satellite

- This ignored the difference in the observation epoch t due to the fact that the two receivers' clocks are unsynchronized

- That is, single difference should be $\Delta\phi(t) = \phi_A(t + \delta t_A) - \phi_B(t + \delta t_B)$ where δts are clock error

Zero-Baseline Receiver Measurements

- If we account for this we have

  $$\Delta\rho(t) \simeq -\vec{b} \cdot \hat{e}(t) + \dot{\rho}(t)\Delta T$$

  with ΔT difference in clock errors

- In the zero-baseline measurement, the two receivers are connected to the same external clock, so the clock variations are identical

- Nevertheless, it is impossible to exactly make the two electrical paths identical, so ΔT is a constant (or very slowly varying)

Zero-Baseline Receiver Measurements

- In the zero-baseline measurement, $\vec{b} = 0$

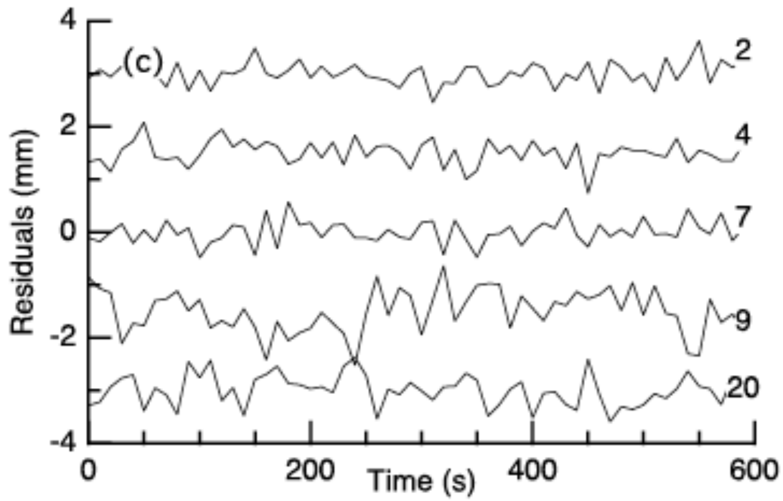
- The single-difference phase between receivers A and B is therefore:
$$\Delta\phi_{AB}(t) \simeq \frac{1}{\lambda}\dot{\rho}(t)\Delta T + \Delta\phi_{\circ} + \Delta\epsilon(t)$$

- $\Delta\phi_{\circ}$ is a constant phase offset that includes the initial ambiguity

- $\Delta\epsilon$ is the difference between observational noise values

Zero-Baseline Receiver Measurements

- Zero-baseline measurements are used to estimate ΔT and $\Delta\phi_\circ$ as well as $\sigma^2_{\Delta\epsilon} = 2\sigma^2_\epsilon$



- Results of *Park et al.* [2004] are (L1) $\sigma_\epsilon \lesssim 0.15$ mm $\simeq 7.4 \times 10^{-4}$ cycles $\simeq 0.27^\circ$

Systematic and Random Errors in GPS

- If $\sigma_\epsilon = 0.2$ mm, then we could estimate (static) site positions with $\sigma < 0.1$ mm from 24 hours of data

- More common to take $\sigma_\epsilon \simeq 10$ mm

- **Systematic** errors occur when model $f(x)$ has errors

- Systematic errors are not necessarily statistical in nature, although we sometimes deal with them as though they were

Assessing systematic errors

- To assess a systematic error $E = f - f_{\text{true}}$, simulate a set of observations using $\Delta y = E$ and perform a least-squares solution

- This will yield $\Delta x(E)$, the error in the parameter adjustments (and hence the parameter estimates) due to E

- Simplified observing scenarios and models can often be used in this simulation

Mapping function errors

- Atmospheric mapping function for elevation angle ε

$$m(\varepsilon) = \frac{1}{\sin\varepsilon + \frac{a}{\sin\varepsilon + \cdots}}$$

with $a \simeq 0.01$

- What is impact of error in a?

$$\Delta m \simeq \frac{\partial m(\varepsilon)}{\partial a}\Delta a \simeq -m(\varepsilon)^2 \left[\frac{\Delta a}{\sin\varepsilon}\right] \simeq -\frac{\Delta a}{\sin^3\varepsilon}$$

- The systematic error will be

$$E(\varepsilon) = -\frac{\tau_\circ^z}{\sin^3\varepsilon}\Delta a \quad \tau_\circ^z = 2.3 \text{ m}$$

Mapping function errors

- Simplified single-difference phase observation equation (prefit residual)

$$\Delta\phi = -\Delta\vec{b}\cdot\hat{e} + \Delta\tau\cdot m(\varepsilon) + \Delta\phi_{\circ}$$

- If we hold one site fixed and estimate other site's position, then expressed in local horizontal coordinates

$$\Delta\vec{b}\cdot\hat{e} = \Delta n \cos A \cos\varepsilon + \Delta e \sin A \cos\varepsilon + \Delta u \sin\varepsilon$$

- For the simulation, we can argue $\Delta n \simeq \Delta e \simeq 0$ due to symmetry of observations

Mapping function errors

- Simplified simulation:

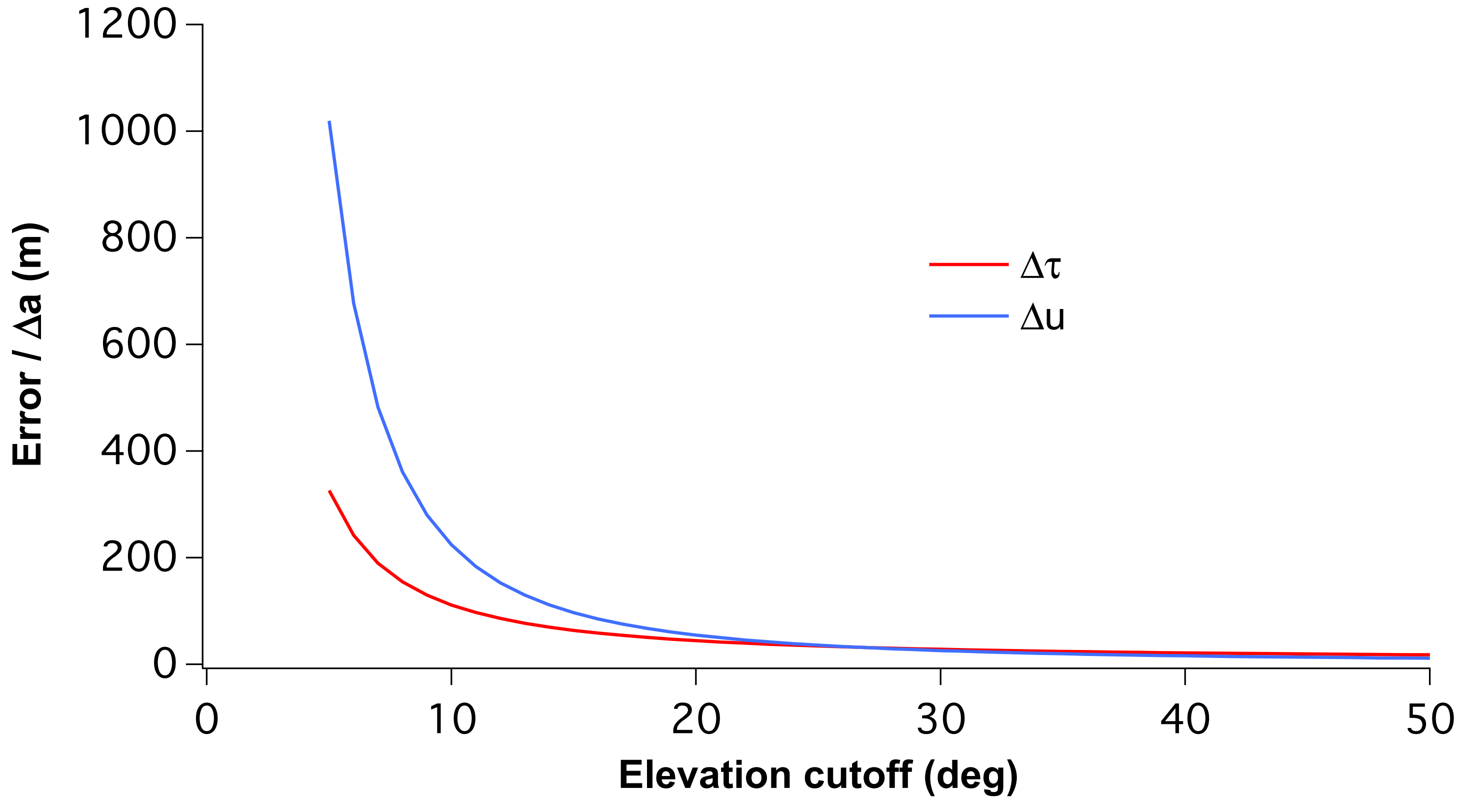
$$-\frac{\tau_\circ^z}{\sin^3\varepsilon}\Delta a = -\Delta u \sin\varepsilon + \Delta\tau \csc\varepsilon + \Delta\phi_\circ$$

- Use least squares to estimate Δu, $\Delta\tau$, and $\Delta\phi_\circ$ for given Δa

- We'll assume we have observations at $\varepsilon = 90^\circ, 89^\circ, \ldots, \varepsilon_{\mathrm{min}}$

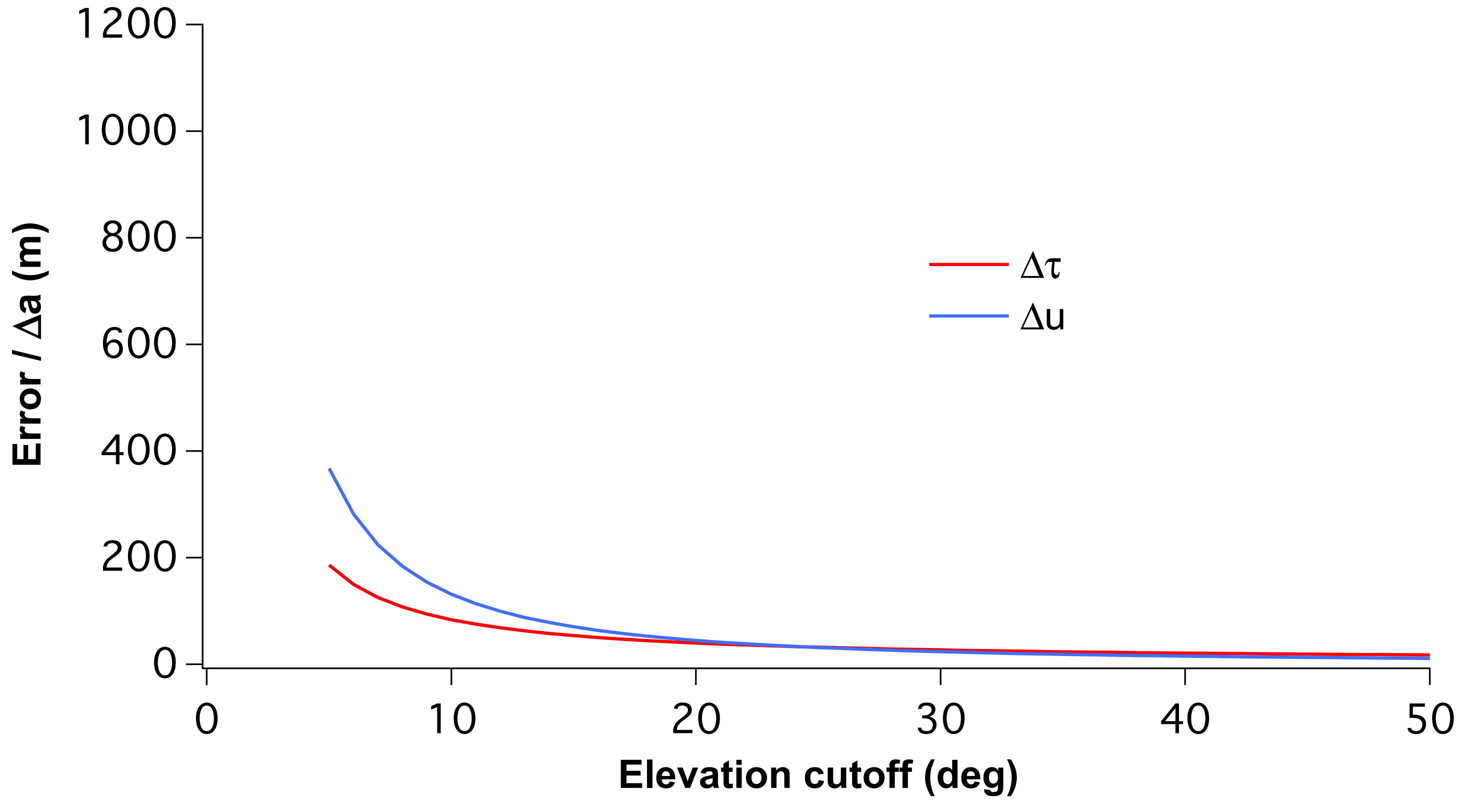
- $\varepsilon_{\mathrm{min}}$ is called the elevation-angle cutoff

Mapping Function Errors



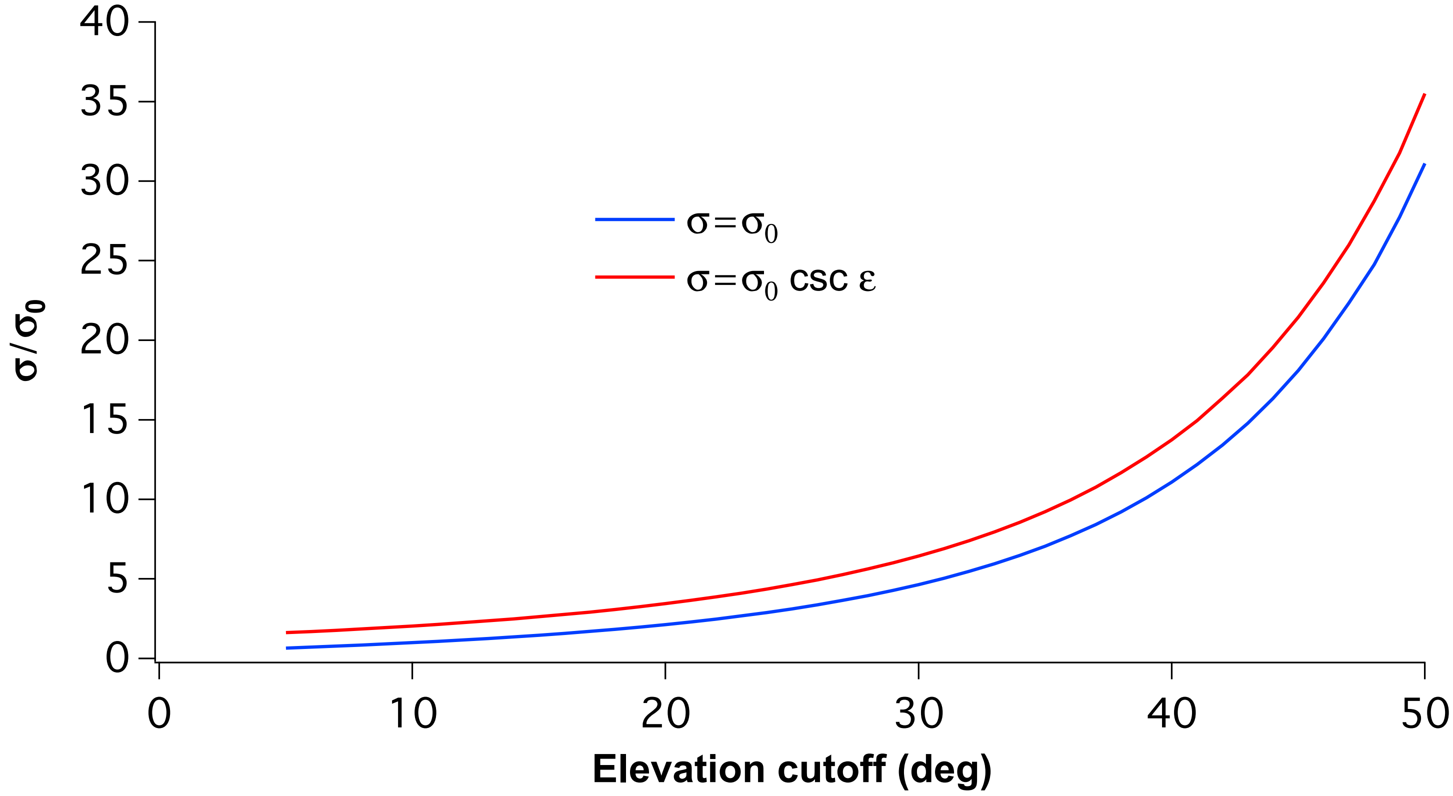
Constant σ_ϵ

Mapping Function Errors



$\sigma_\epsilon \sim \csc \varepsilon$

Impact of Elevation Weighting on Standard Deviations



Antenna Directionality

Some applications require antennas with significant directivity

Green Bank Telescope (NRAO)

Antenna Gain Pattern (High Directivity)

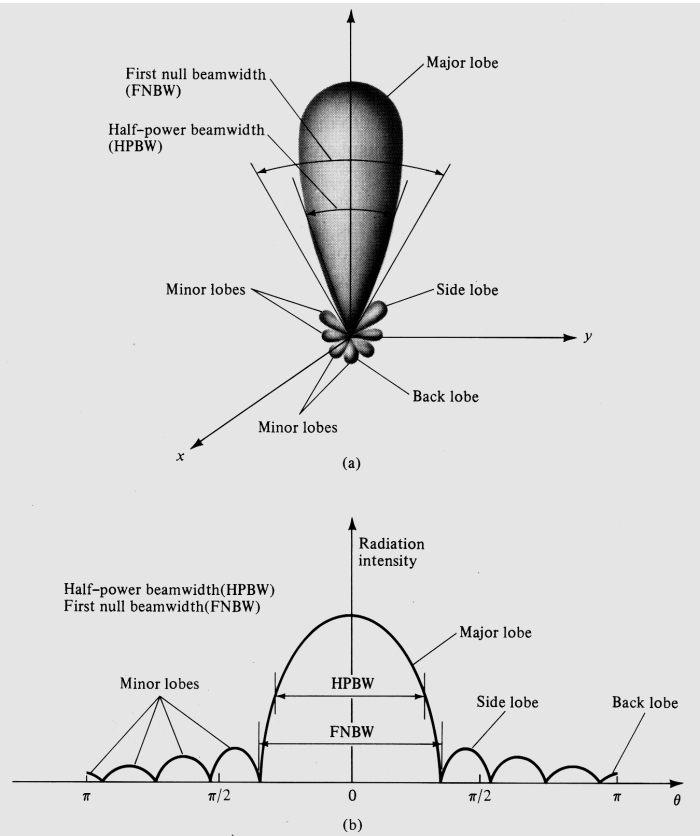
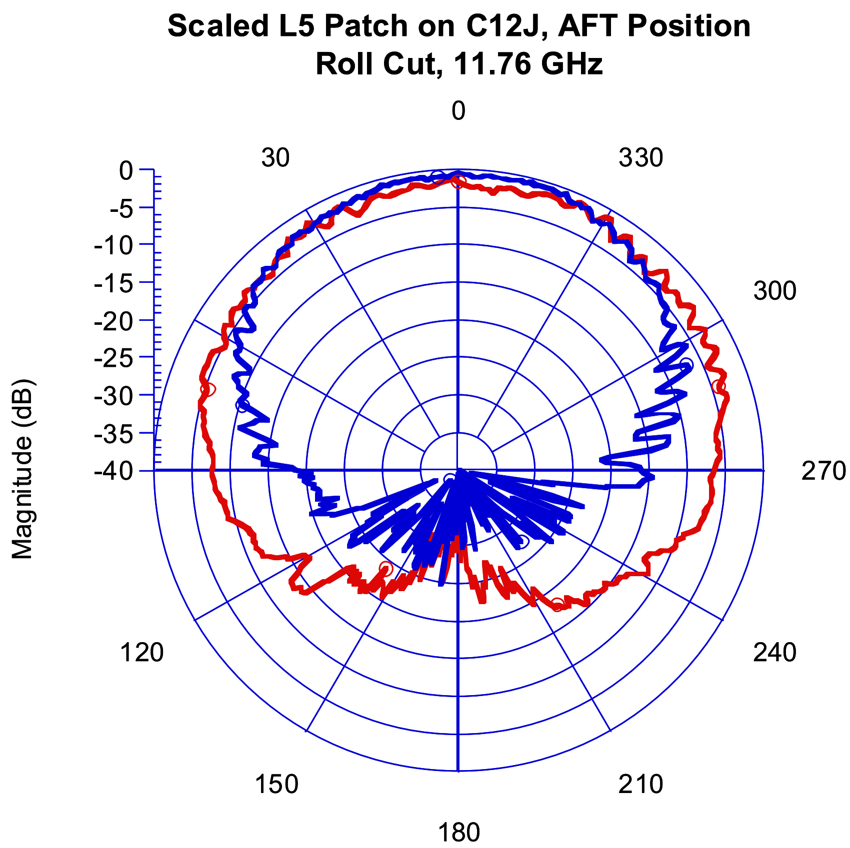
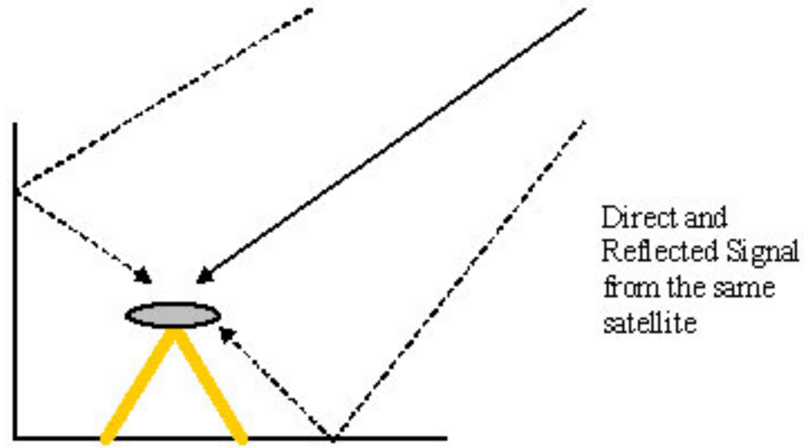


Figure 2.4 (a) Radiation lobes and beamwidths of an antenna pattern. (b) Linear plot of power pattern and its associated lobes and beamwidths.

GPS Antenna Gain Pattern
(little directivity)

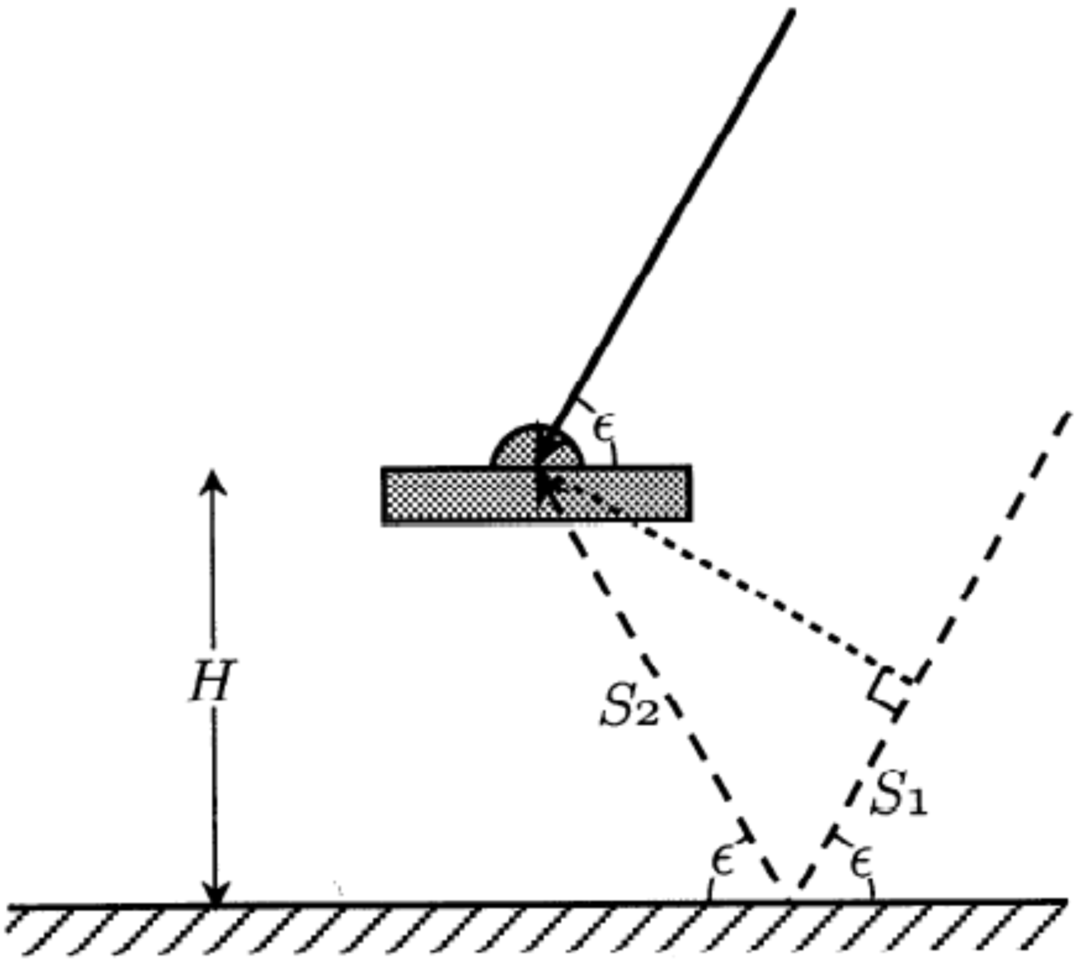


Multipath



http://earthmeasurement.com

Phase Multipath Geometry



Phase Multipath

- Consider signal at antenna as sum of direct (unreflected) and reflected signals:

$$S(t) = U(t) + \alpha R(t)$$

  α accounts for gain and reflectance

- Unreflected signal is

$$U(t) = U_\circ e^{-2\pi f t}$$

- Reflected signal is

$$R(t) = U\left(t - \frac{S_1 + S_2}{c}\right)$$

Phase Multipath

- Then total signal is

$$S(t) = U_\circ e^{-2\pi f t}\left(1 + \alpha e^{2\pi i (S_1 + S_2)/\lambda}\right)$$

- Write as change in amplitude and phase wrt $U(t)$

$$S(t) = \beta U(t) e^{i\delta\phi}$$

- Then

$$\tan\delta\phi = \frac{\alpha \sin\psi(\varepsilon)}{1 + \alpha\cos\psi(\varepsilon)} \qquad \beta^2 = 1 + \alpha + 2\alpha\cos\psi(\varepsilon)$$

with $\psi(\varepsilon) = 4\pi(H/\lambda)\sin\varepsilon$

Phase Multipath

$$\tan \delta\phi = \frac{\alpha \sin \psi(\varepsilon)}{1 + \alpha \cos \psi(\varepsilon)} \quad \beta^2 = 1 + \alpha^2 + 2\alpha \cos \psi(\varepsilon)$$

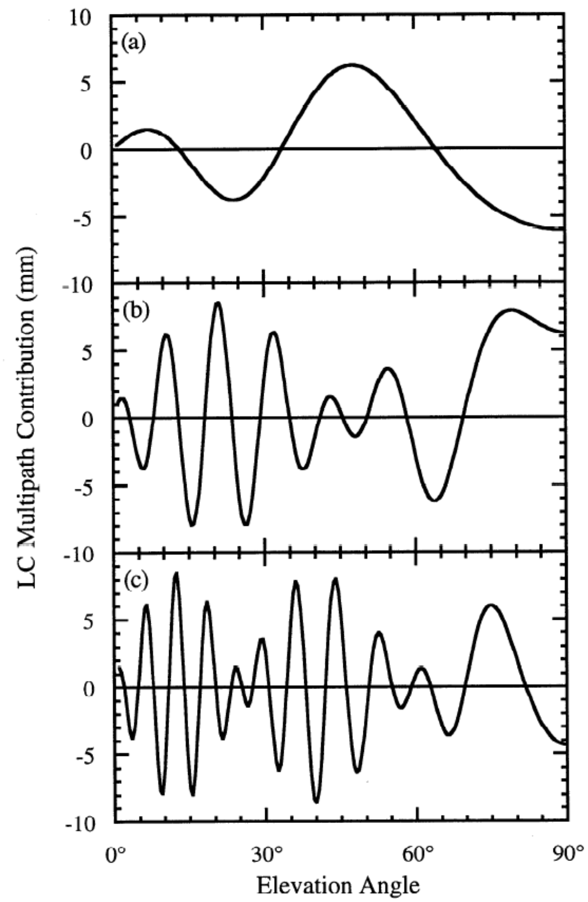
$$\psi(\varepsilon) = 4\pi(H/\lambda) \sin \varepsilon$$

- Multipath affects both phase and amplitude (i.e., SNR) of received signal

- Although multipath represents an electrical path length *increase*, because it comes in as a phasor $\delta\phi$ may be positive or negative

- As $\alpha \to 0$, $\delta\phi \to 0$ and $\beta \to 1$

LC Phase Multipath



(a) $H = 0.15$ m; (b) $H = 0.6$ m; (c) $H = 1$ m

GPS Multipath

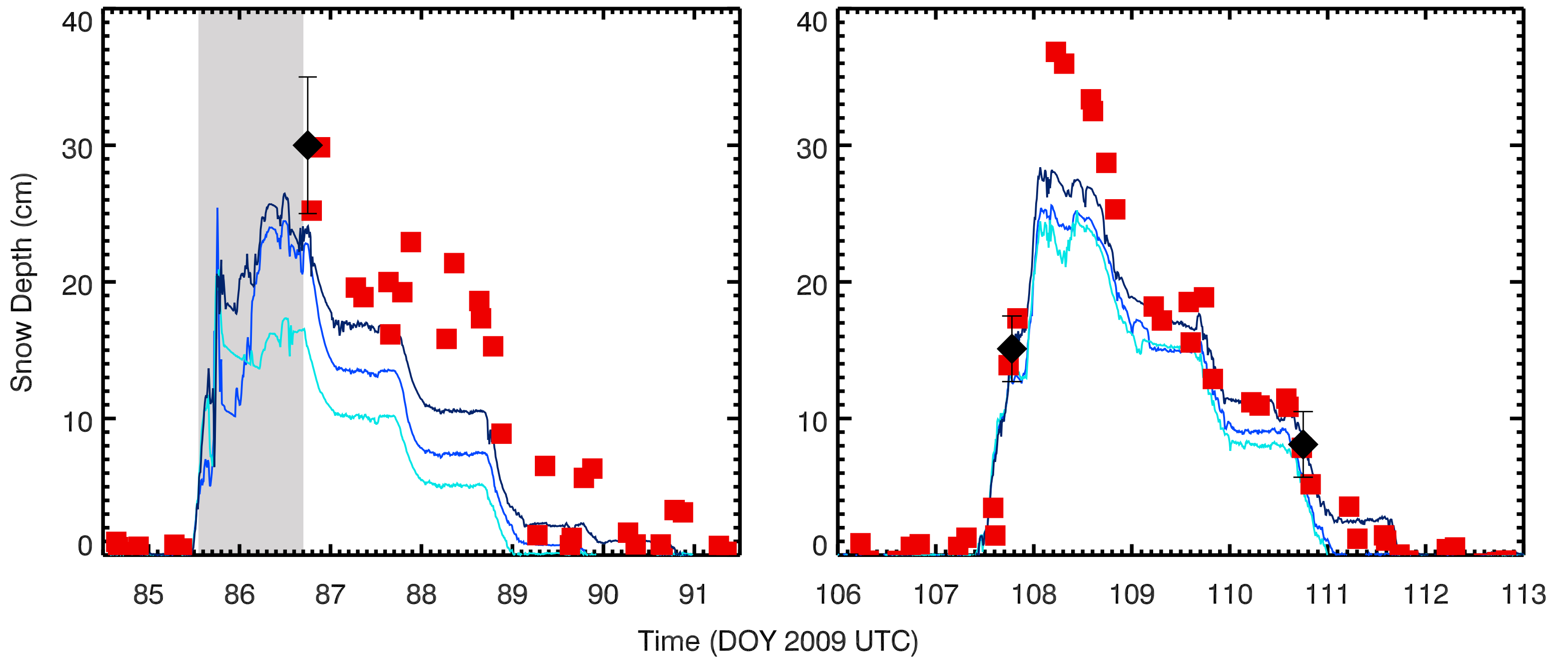
- Except for simple cases, attempts to model multipath in GPS have not been successful

- Multipath remains one of the most significant error sources

- Silver lining: The amplitude of the multipath, through the reflectance α, tells us something about the reflector

- GPS is being used to estimate soil moisture, ground snow, and vegetation

Snow depth from SNR



Larson et al. [2009]

GPS Multipath: Sidereal filtering

- Multipath error repeats with $\sim$1 sidereal day repeat time for satellite geometry

- Error M_x in position x due to multipath:

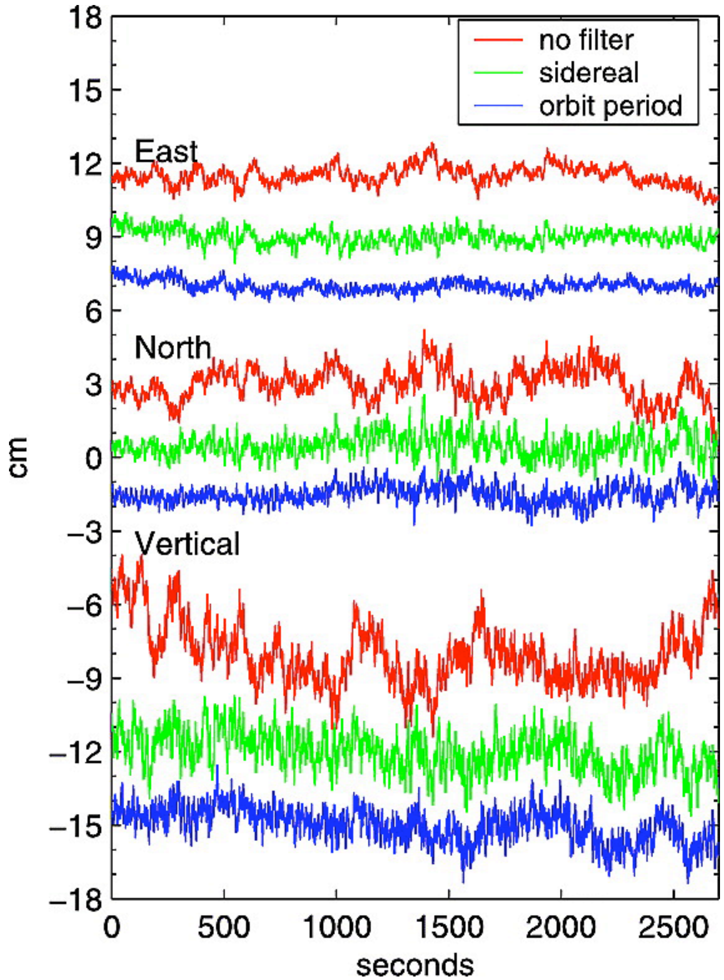
$$M_x(t) = \frac{1}{2}\Big[x(t) + x(t + 1 \text{ day})\Big]$$

- Position after sidereal filtering:

$$x_s(t) = x(t) - M_x(t)$$

- This technique is used in GPS seismology

Sidereal filtering



Choi et al. [2004]

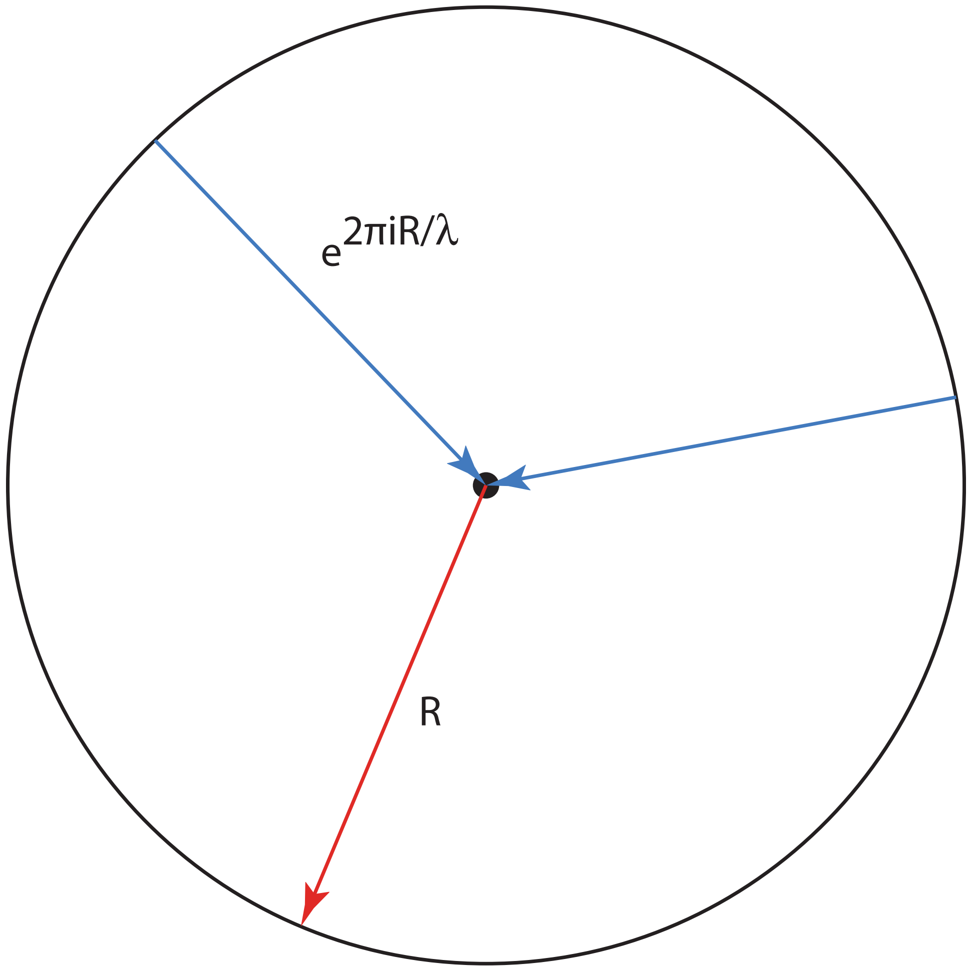
Phase-Center Variations

- A perfect, isotropic transmitting antenna would radiate energy in a vacuum such that surfaces of equal phase were spheres centered at a point

- This point is called the **phase center**

- If such an antenna were used to receive signals, then signals emanating from different points on a sphere would all be received with the same phase

Phase Center

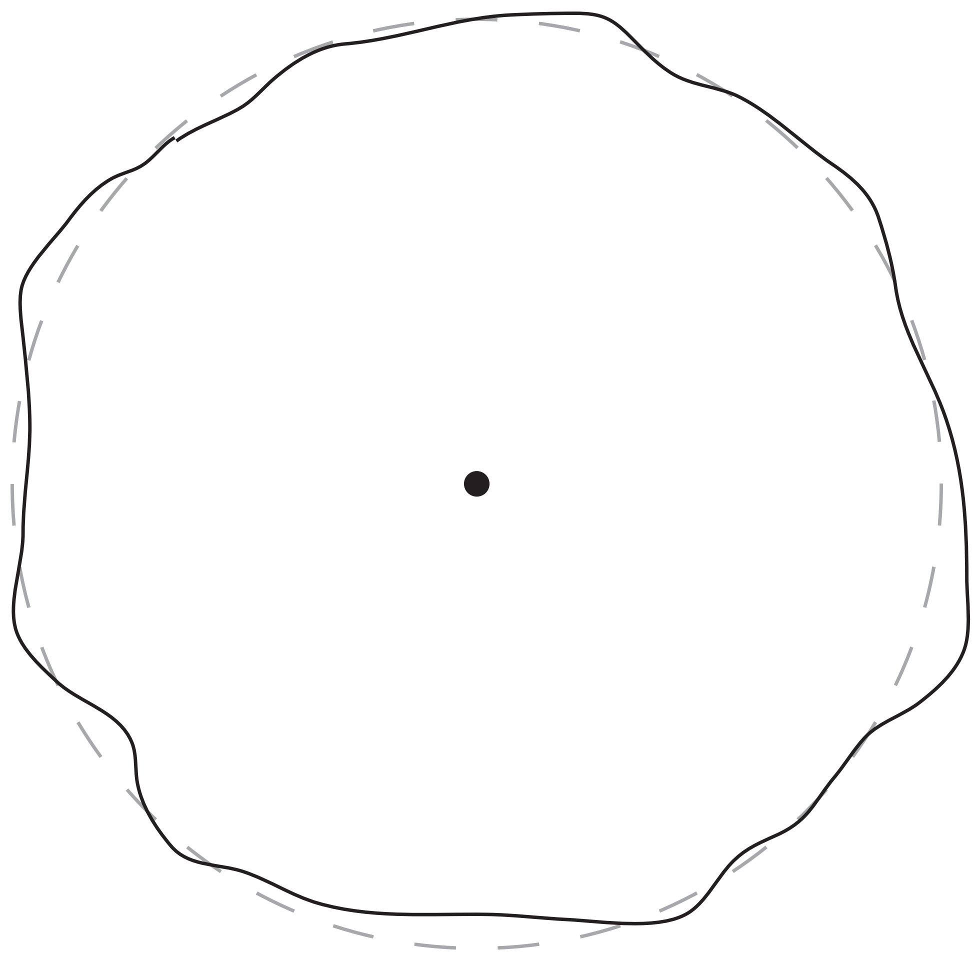


Phase-Center Variations

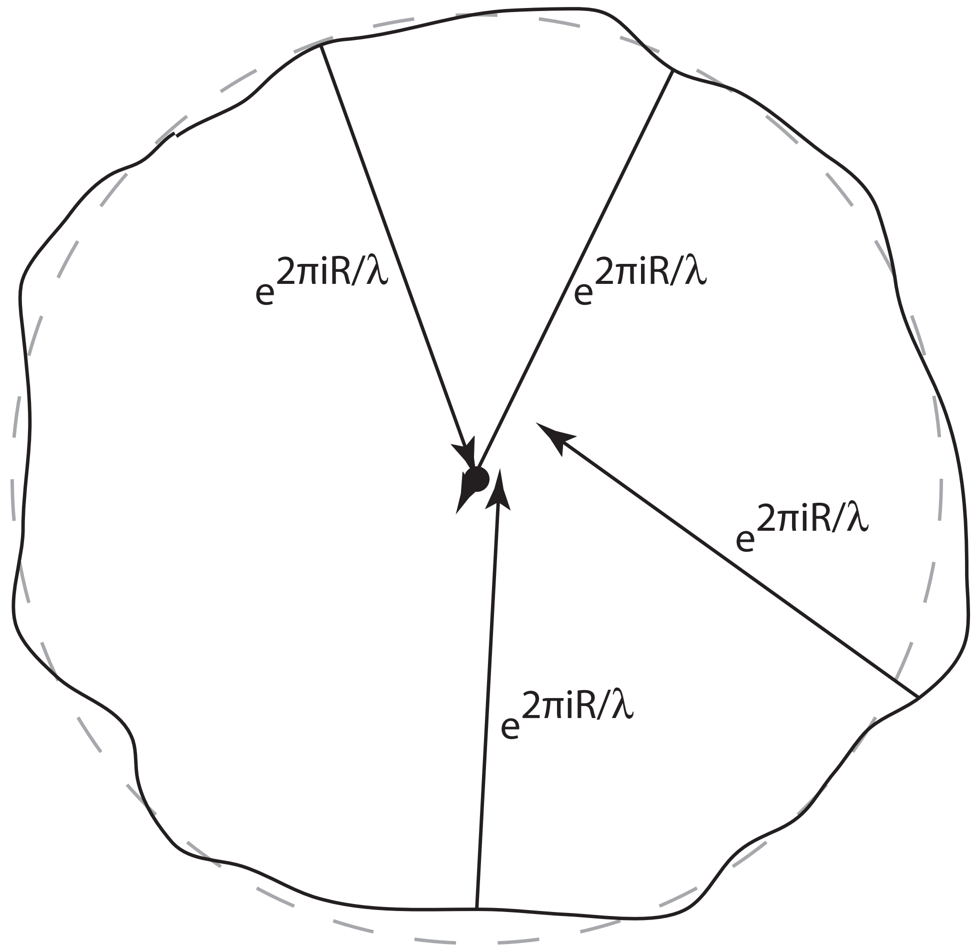
- For an imperfect (i.e., realistic) transmitting antenna, the surfaces of equal phase are not spheres

- In effect, there is not a single point that can be called the phase center, and the phase-center is said to vary with direction

Realistic Phase Front



Realistic Phase Front



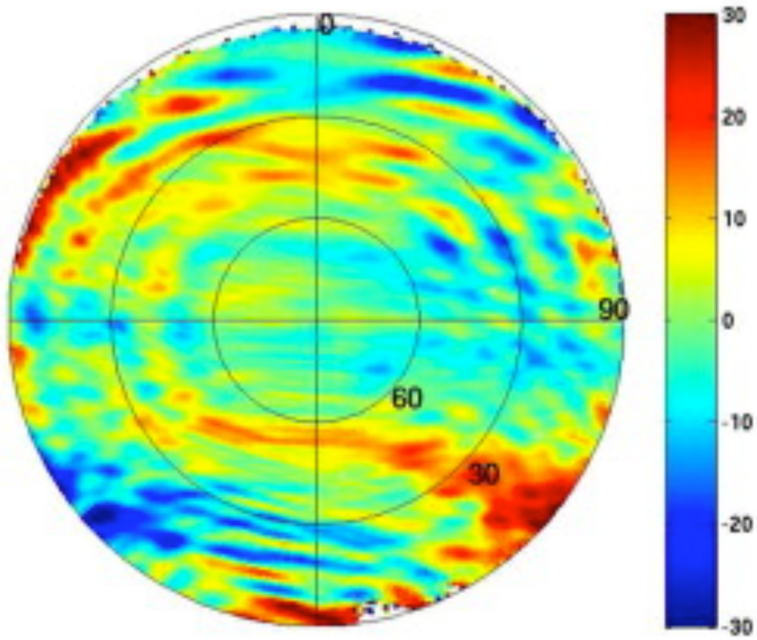
Phase-Center Variations

- A single phase center is what we'd like with GPS so that we can related the change of phase to the distance the signal has traveled

- For a GPS antenna receiving a signal from azimuth A and elevation ε there is a contribution to the phase $\delta\phi_{pc}(A, \varepsilon)$ due to the phase-center variation

- The phase-center variation depends on frequency (i.e., L1 is different than L2)

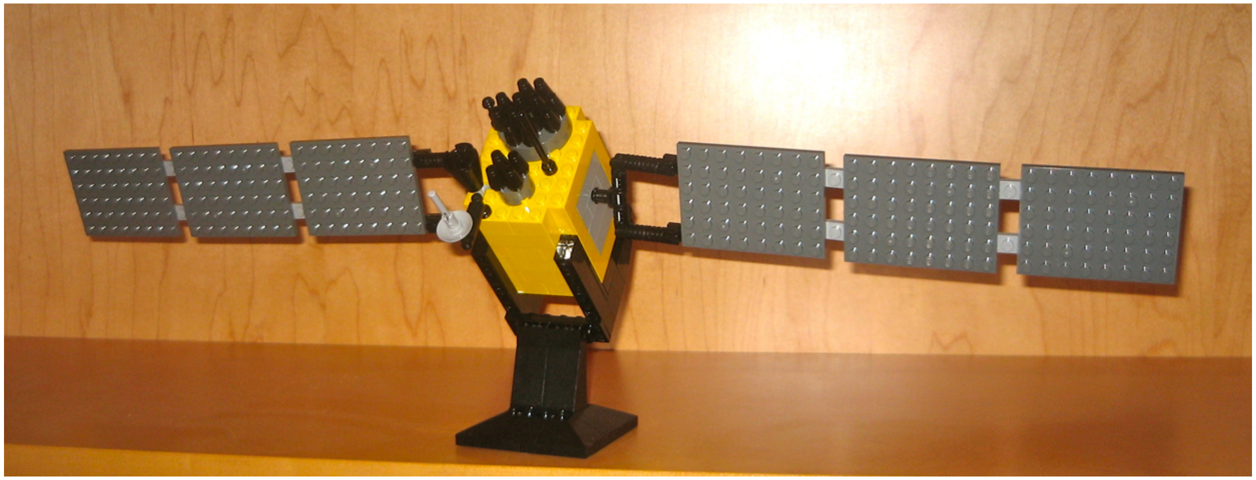
Example PCVs (LC)



(c) Final 1°x 1° PCV map [mm]

Bock et al. [2011]

GPS Satellite Model Errors



- GPS satellite phase center offset (i.e., nominal phase center is different from center of mass)
- GPS satellite phase-center variations
- Solar radiation

Other sources of systematic errors

- Signal scattering from antenna mount
- Snow and ice on antenna
- These all impact vertical more than horizontal

GPS position accuracy: Summary

- The "accuracy" of GPS is highly dependent on the application, meaning:
  - Timespan of data used to make estimates
  - Accessibility to reference frame
  - Kinematics of antenna
  - Data combination (e.g., accelerometer)
  - Analysis methods

GPS position accuracy: Summary

- "Accuracy" has more than one meaning in literature

  1. How accurate is position estimate at a particular epoch?

  2. How accurate are estimates of, e.g., deformation parameters

  3. Repeatability vs. "truth" (comparisons of results from multiple geodetic measurement systems)

GPS position accuracy: Summary

- Improvements in accuracy over last 2–3 decades:

  1. IGS leads to better clocks and orbits

  2. Filling out constellation

  3. Improvements in models, analysis techniques, bias fixing

  4. Continuity of observations

  5. Improvements/standardization of tracking equipment