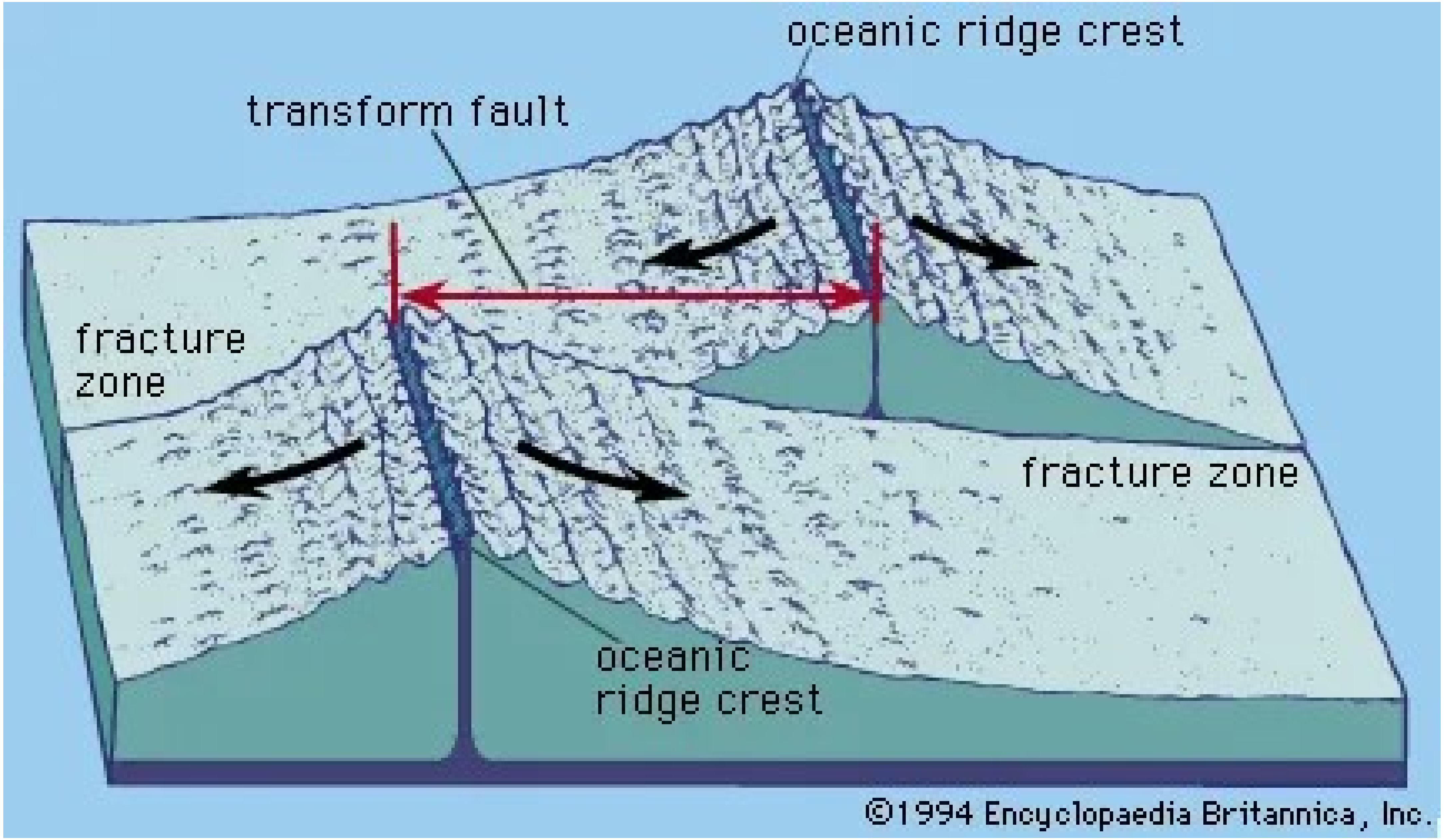
Solid Earth Dynamics

Bill Menke, Instructor

Lecture 4

Today:

Depth - Age


oceanic ridge crest
transform fault
fracture zone
fracture zone
oceanic ridge crest
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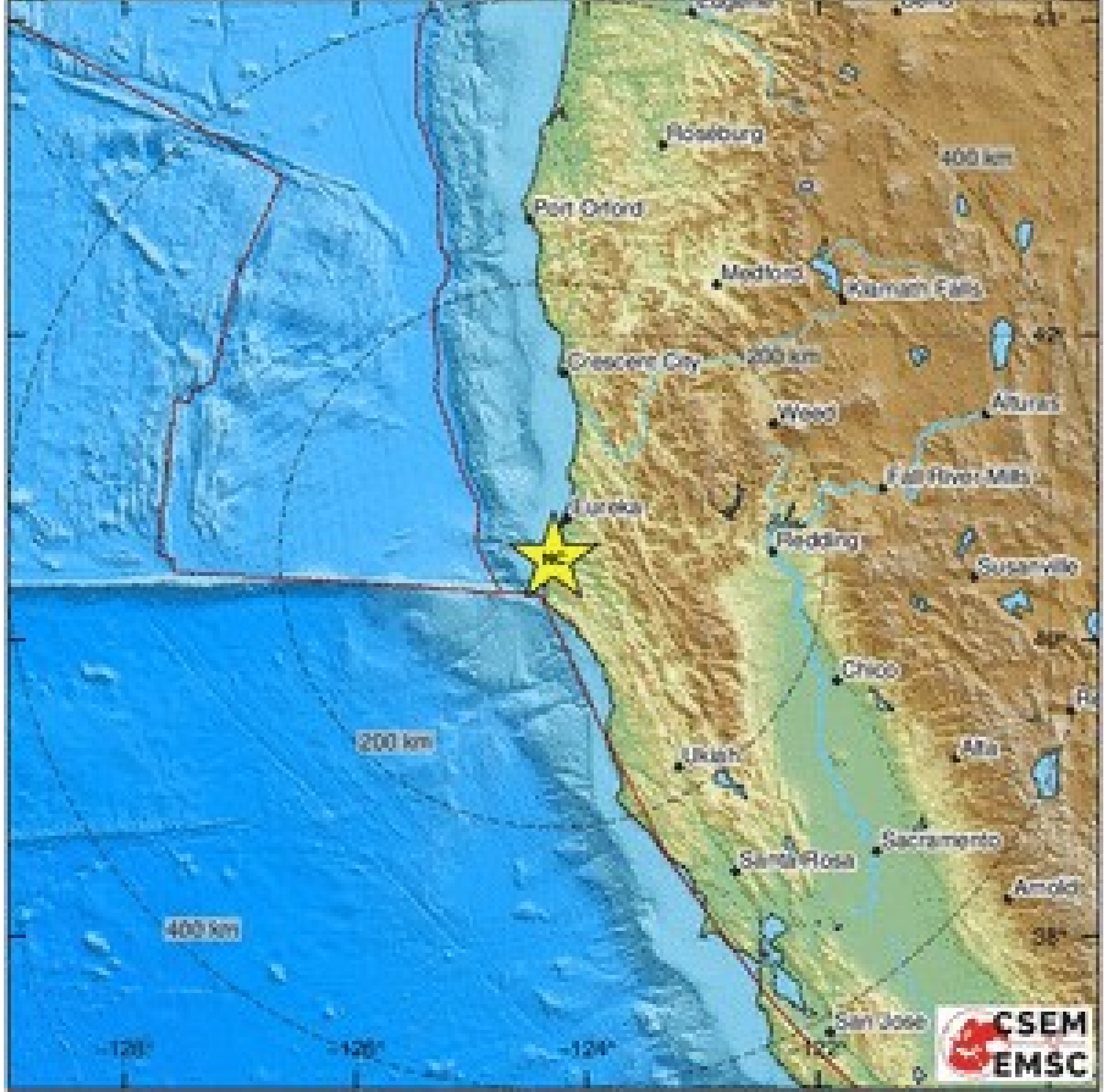
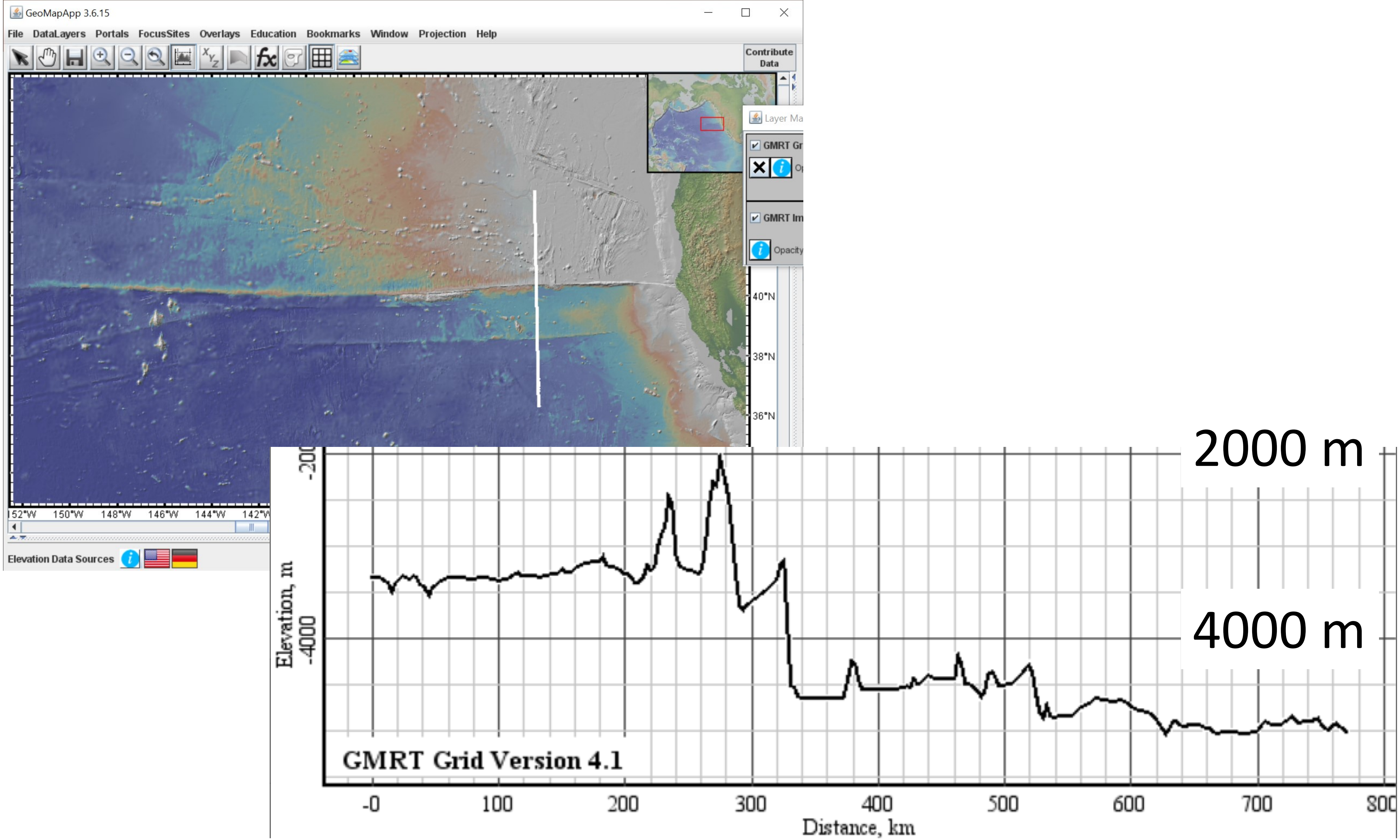
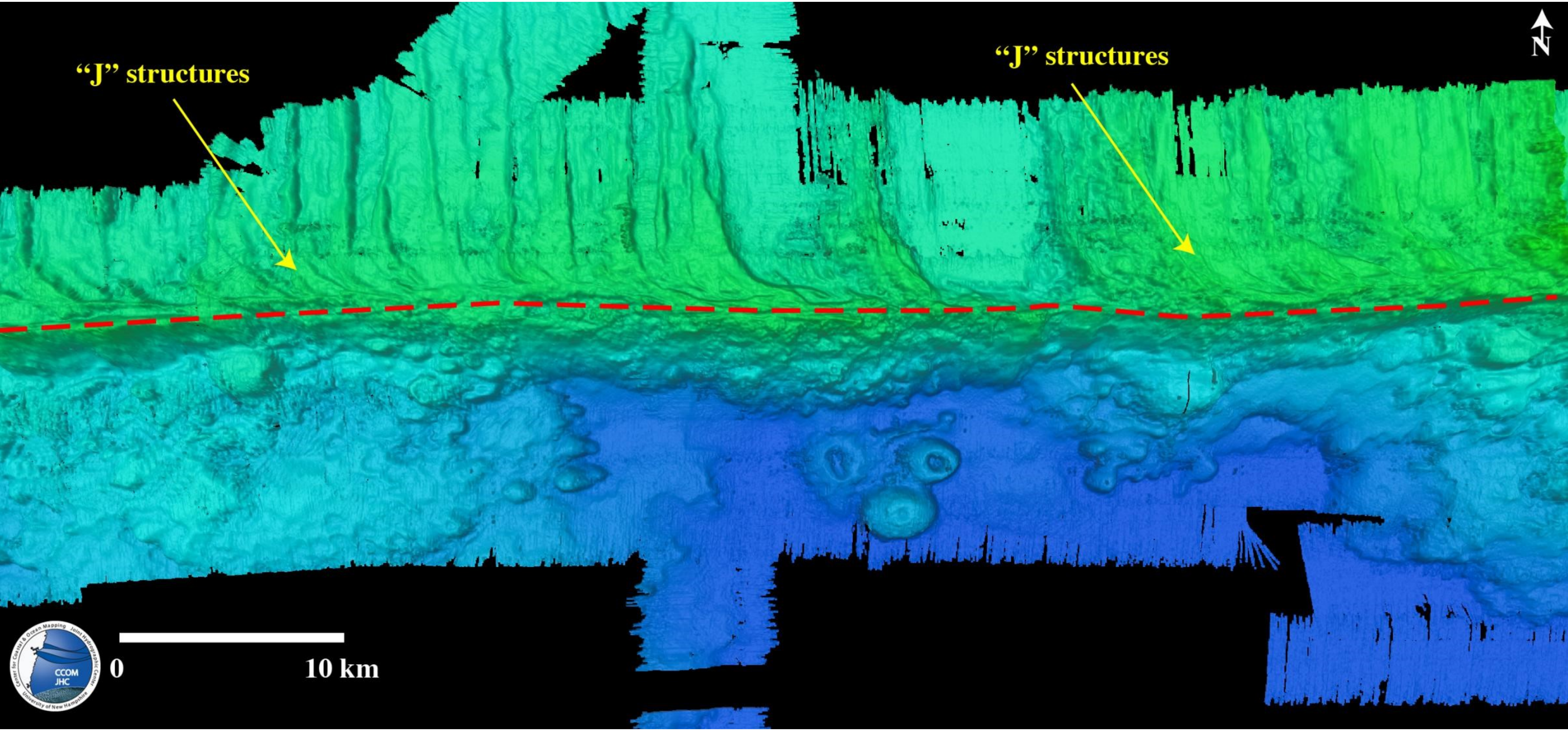
Mendicino
fracture zone

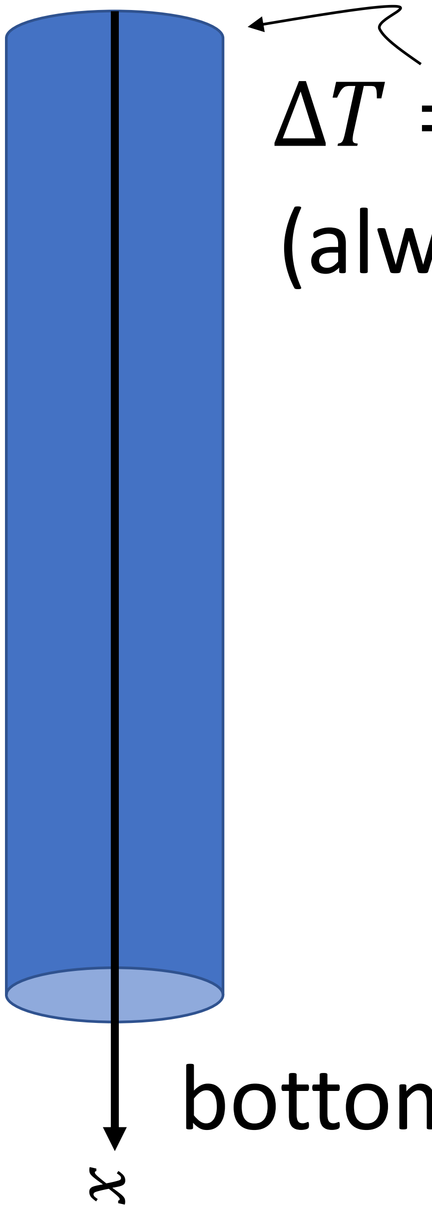



GeoMapApp 3.6.15
File DataLayers Portals FocusSites Overlays Education Bookmarks Window Projection Help
Contribute Data
Layer Ma
GMRT Gr
GMRT Im
Opacity
40°N
38°N
36°N
152°W 150°W 148°W 146°W 144°W 142°W
Elevation Data Sources
2000 m
4000 m
-2000
-4000
Elevation, m
GMRT Grid Version 4.1
-0 100 200 300 400 500 600 700 800
Distance, km


"J" structures
"J" structures
N
0
10 km
CCOM
JHC
Center for Coastal & Ocean Mapping
Joint Hydrographic Center
University of New Hampshire

Back to the rod

Top = surface of earth



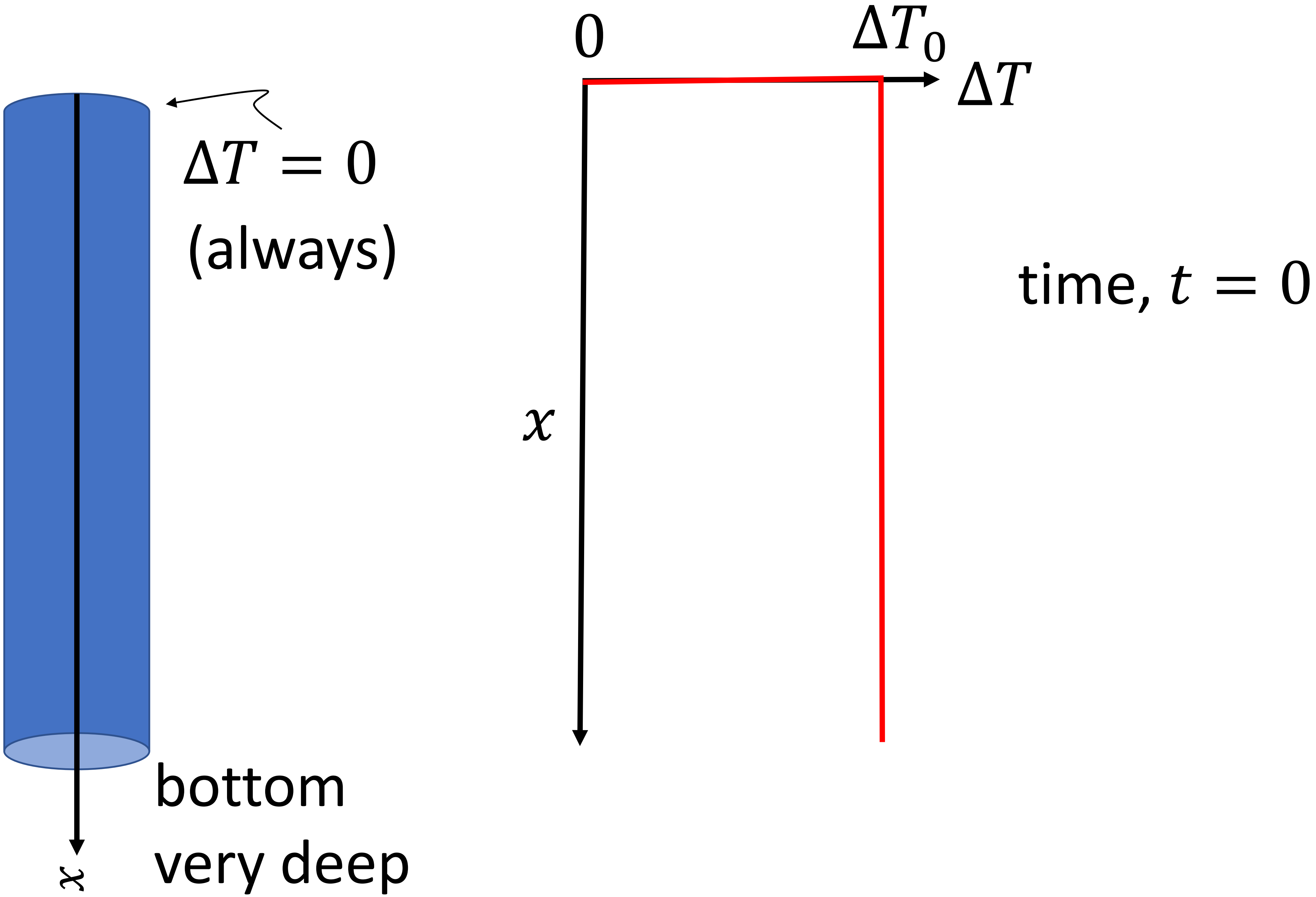
$\Delta T = 0$

(always)

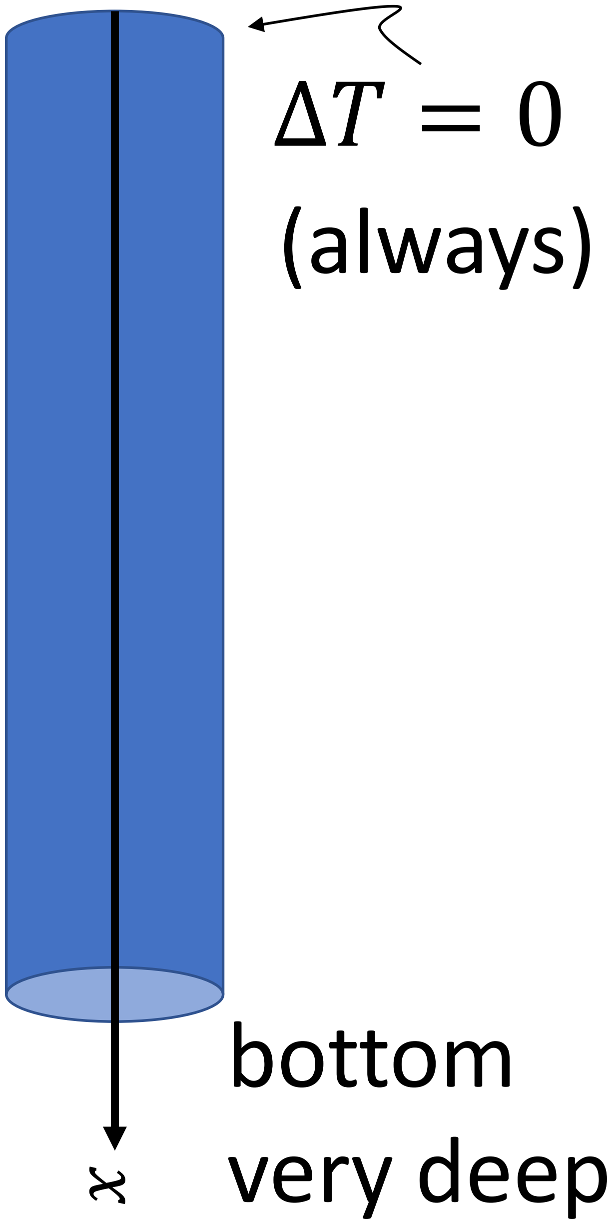
bottom very deep

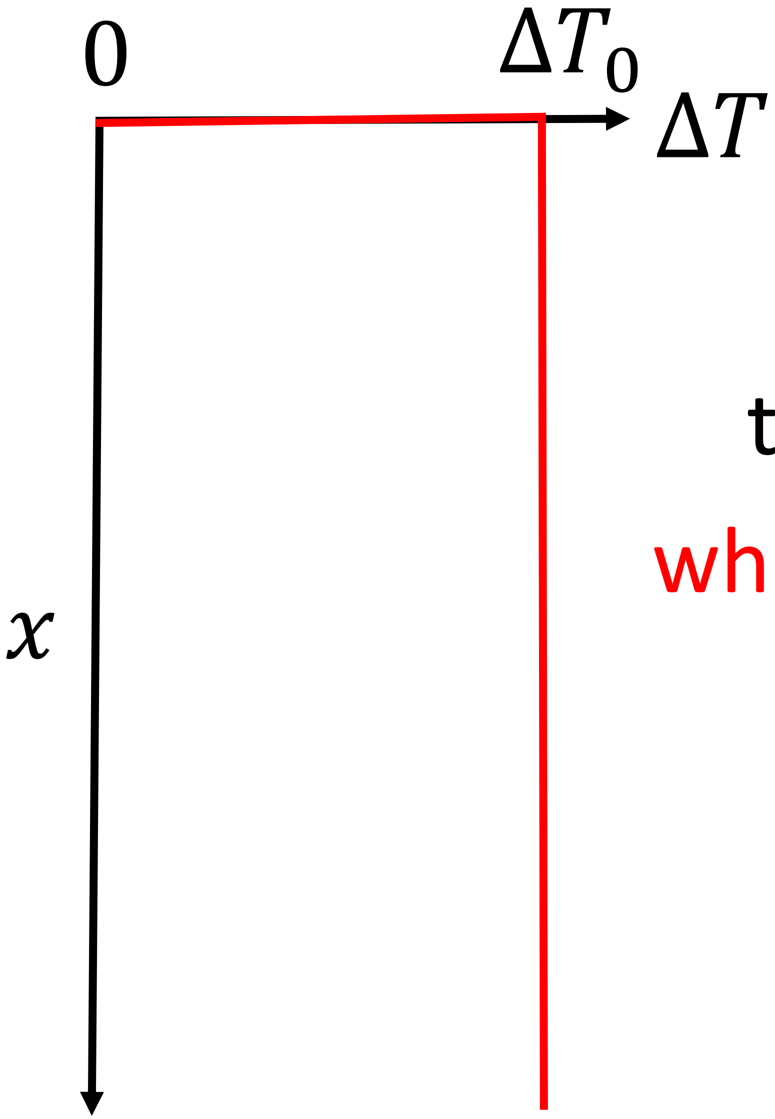
$$\rho c_p \frac{d\Delta T}{dt} = k \frac{d^2 \Delta T}{dx^2}$$

$$\kappa = \frac{k}{\rho c_p}$$



$\Delta T = 0$
(always)
bottom
very deep
x
0
ΔT_0
ΔT
x
time, $t = 0$





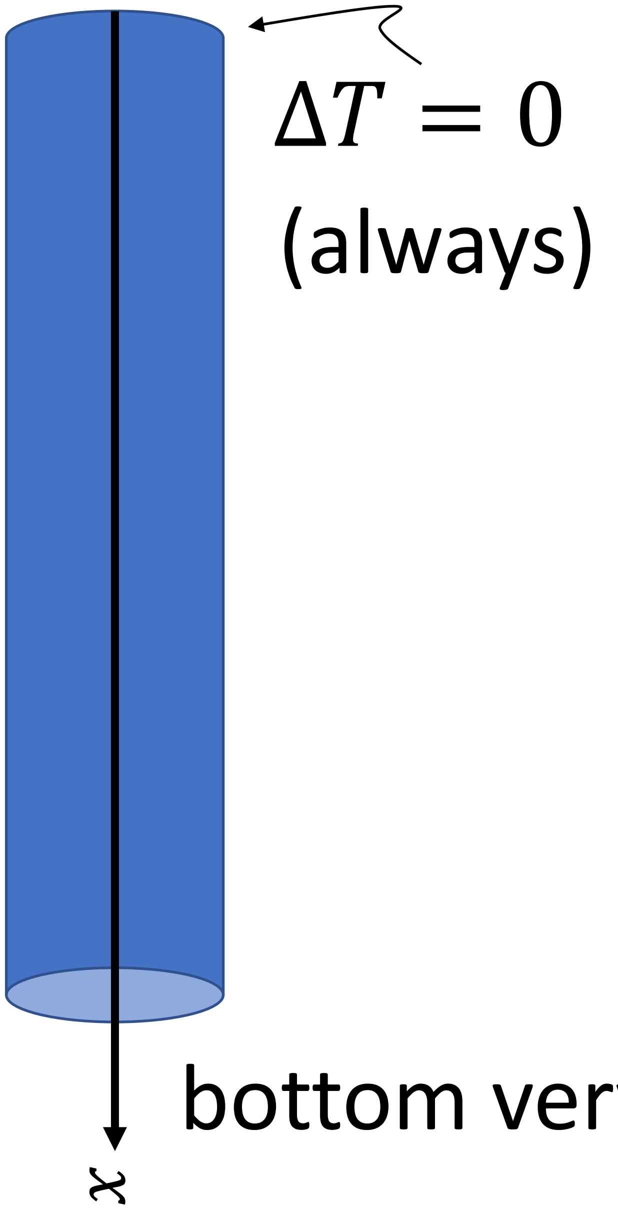
time, $t = 0$

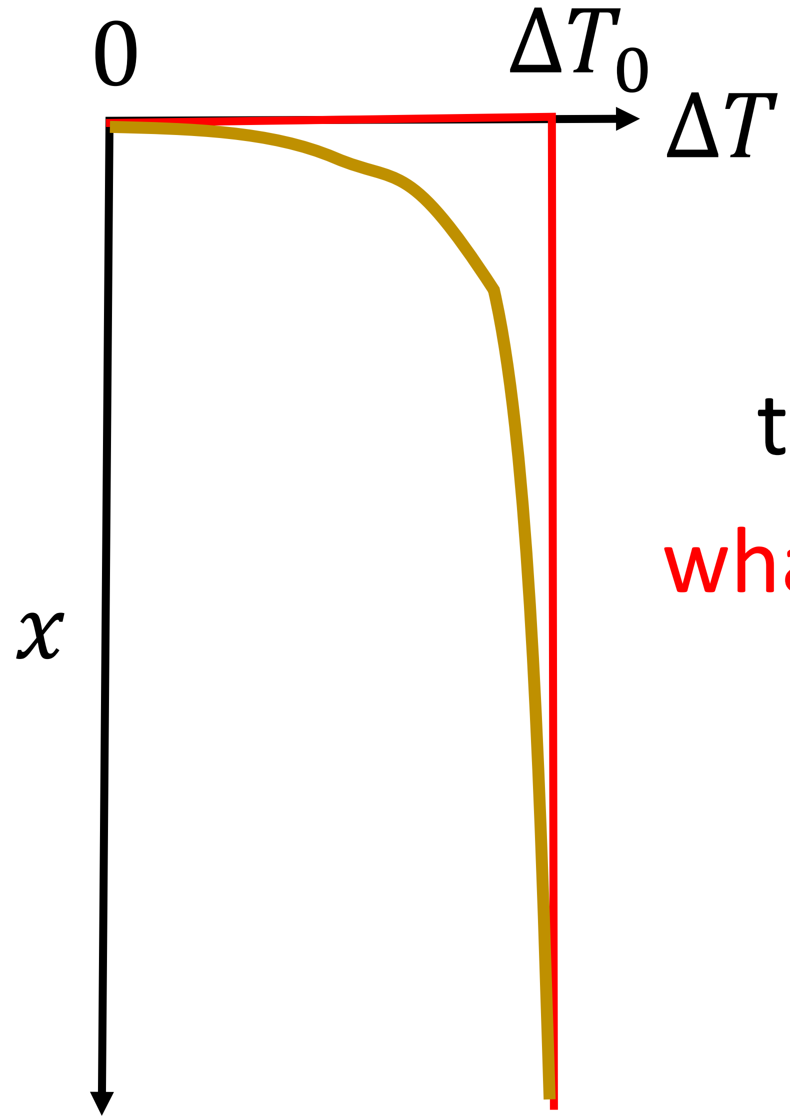
what happens next?

at $x = 0$?

just below 0?

very deep?





time, $t = 0$

what happens next?

at $x = 0$?

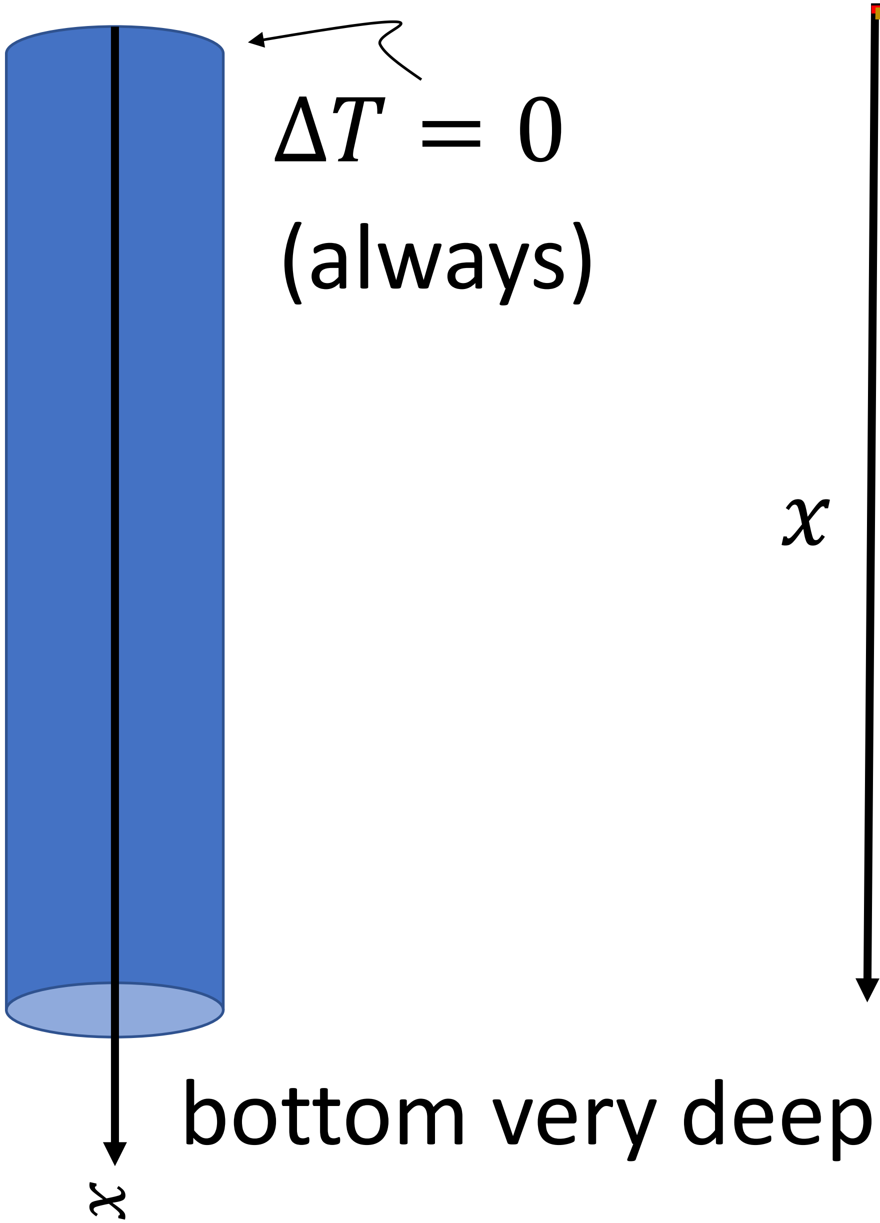
stays 0?

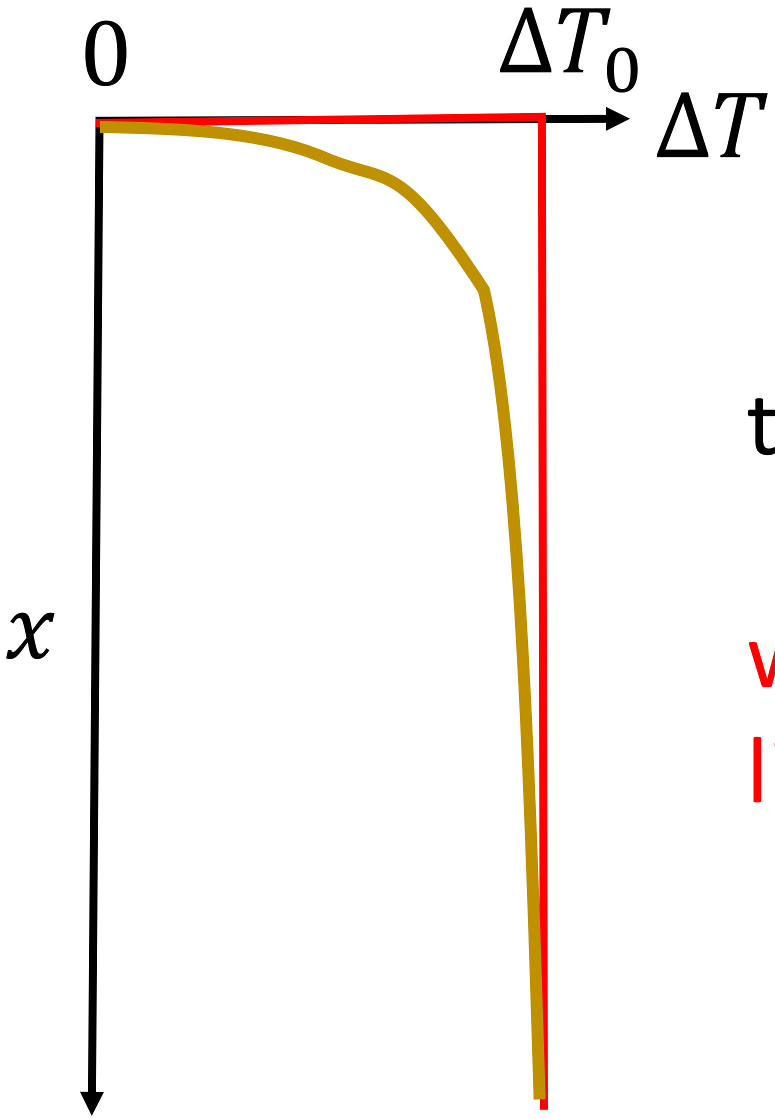
just below 0?

strong cooling

very deep?

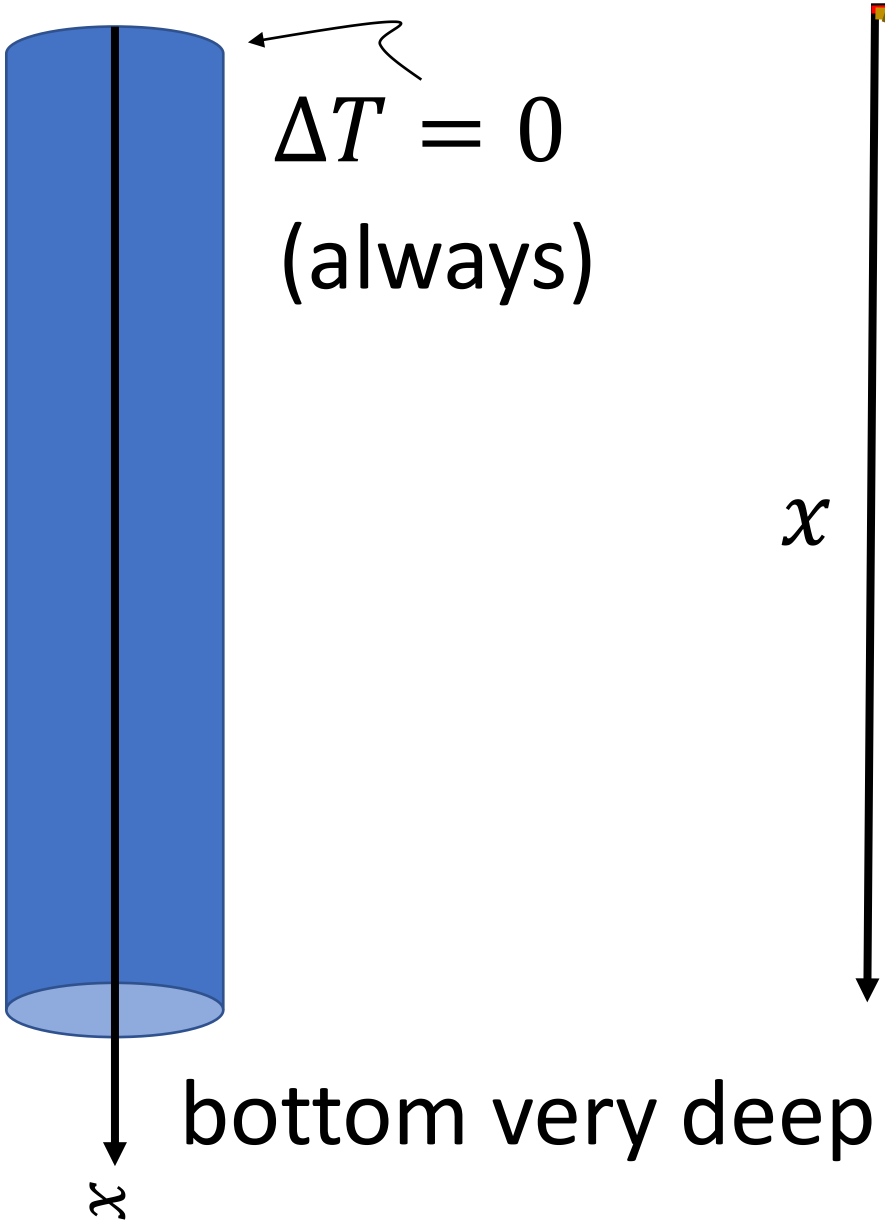
stays close to ΔT_0

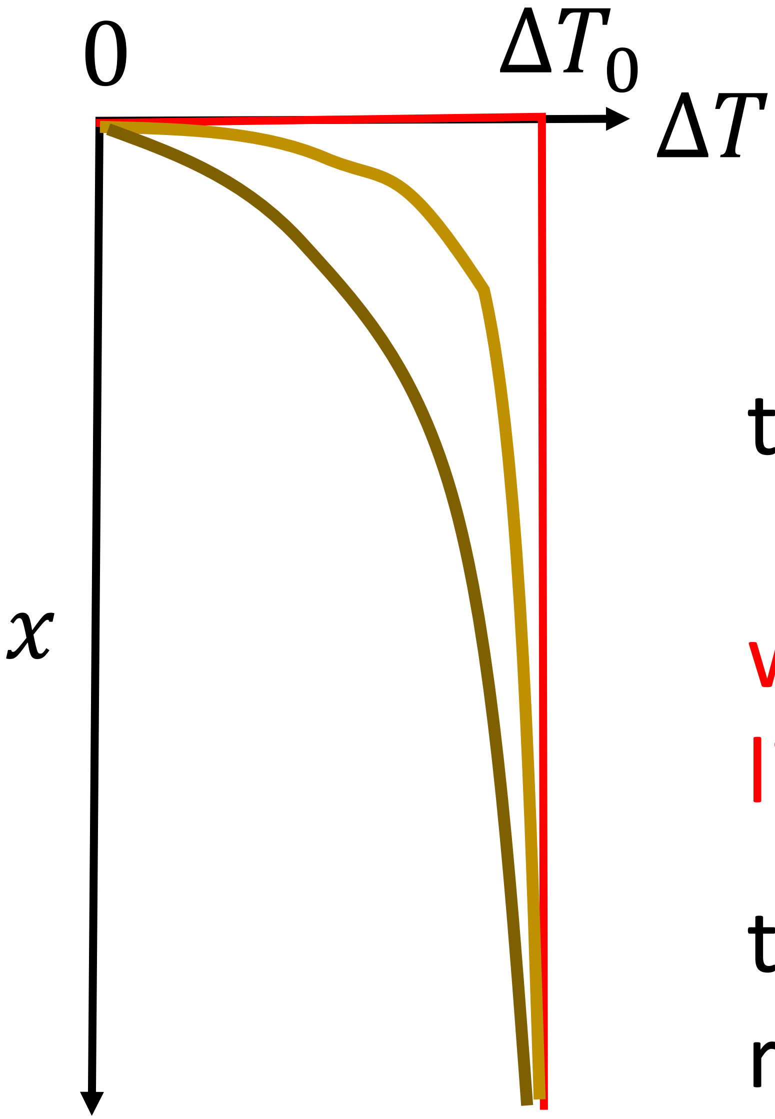




time, $t = 0$

what happens a little later?





time, $t = 0$

what happens a little later?

thickness of cooled region increases

solution

$$\Delta T(x,t) = \Delta T_0 \ \mathrm{erf}\left\{\frac{x}{\sqrt{4\kappa t}}\right\}$$

the error function

named function

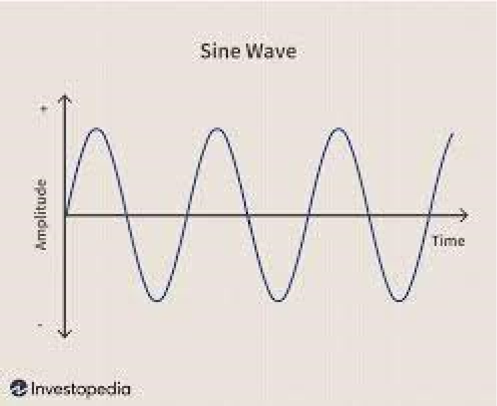
useful properties, more complicated than polynomials

which are the ones you study in trigonometry?

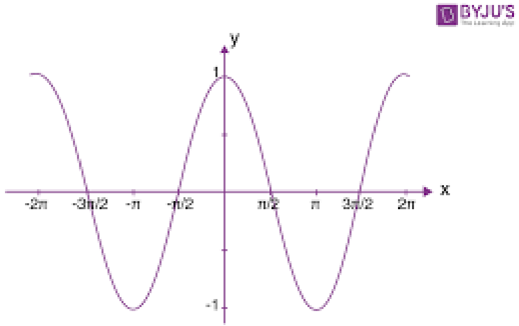
named function

useful properties, more complicated than polynomials

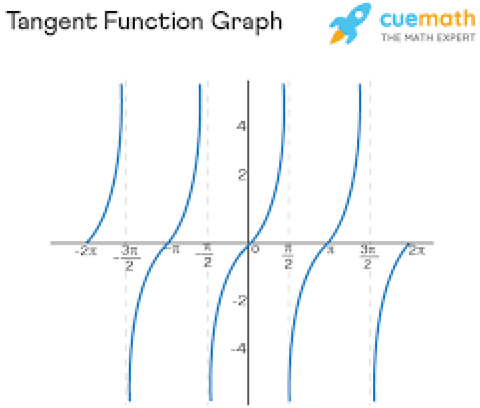
$\sin(x)$



$\cos(x)$



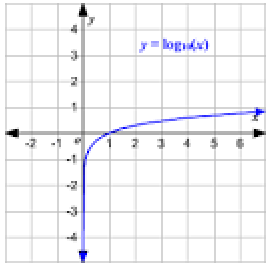
$\tan(x)$



here are some others you've probably used

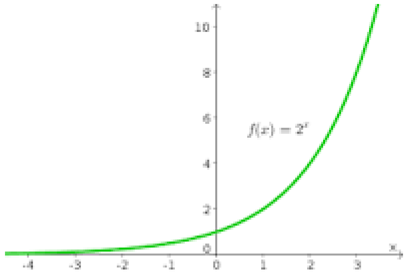
logarithmic function

$$\ln(x)$$



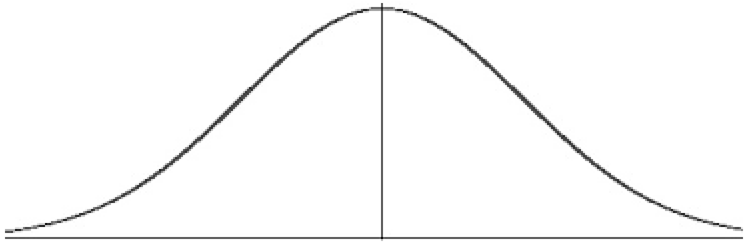
exponential function

$$\exp(x)$$



Normal or Gaussian function

$$\mathrm{N}(x)$$



have you encountered any others?

have you encountered any others?

Anger function

Bessel Function

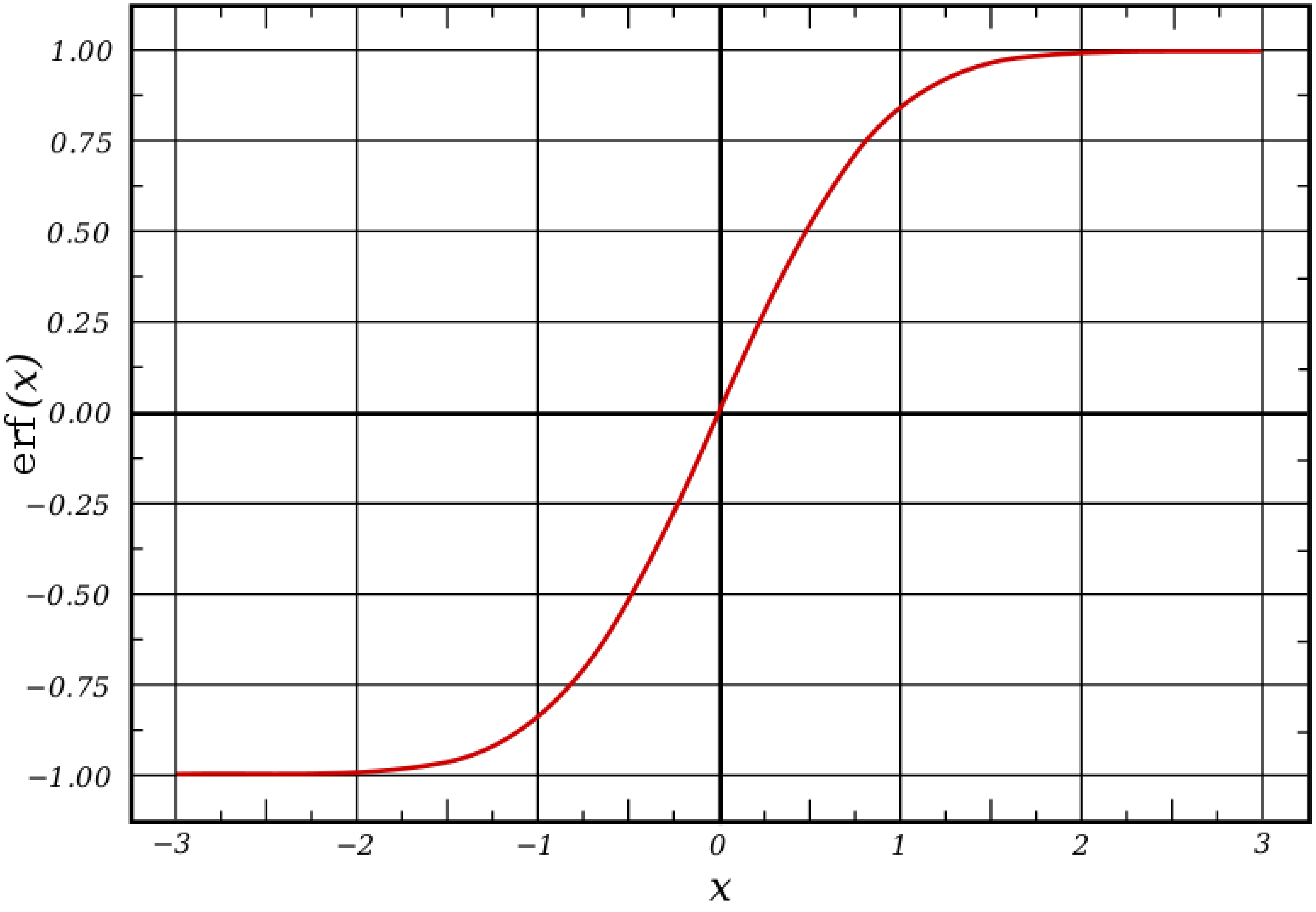
Chi-squared function

Dawson Function

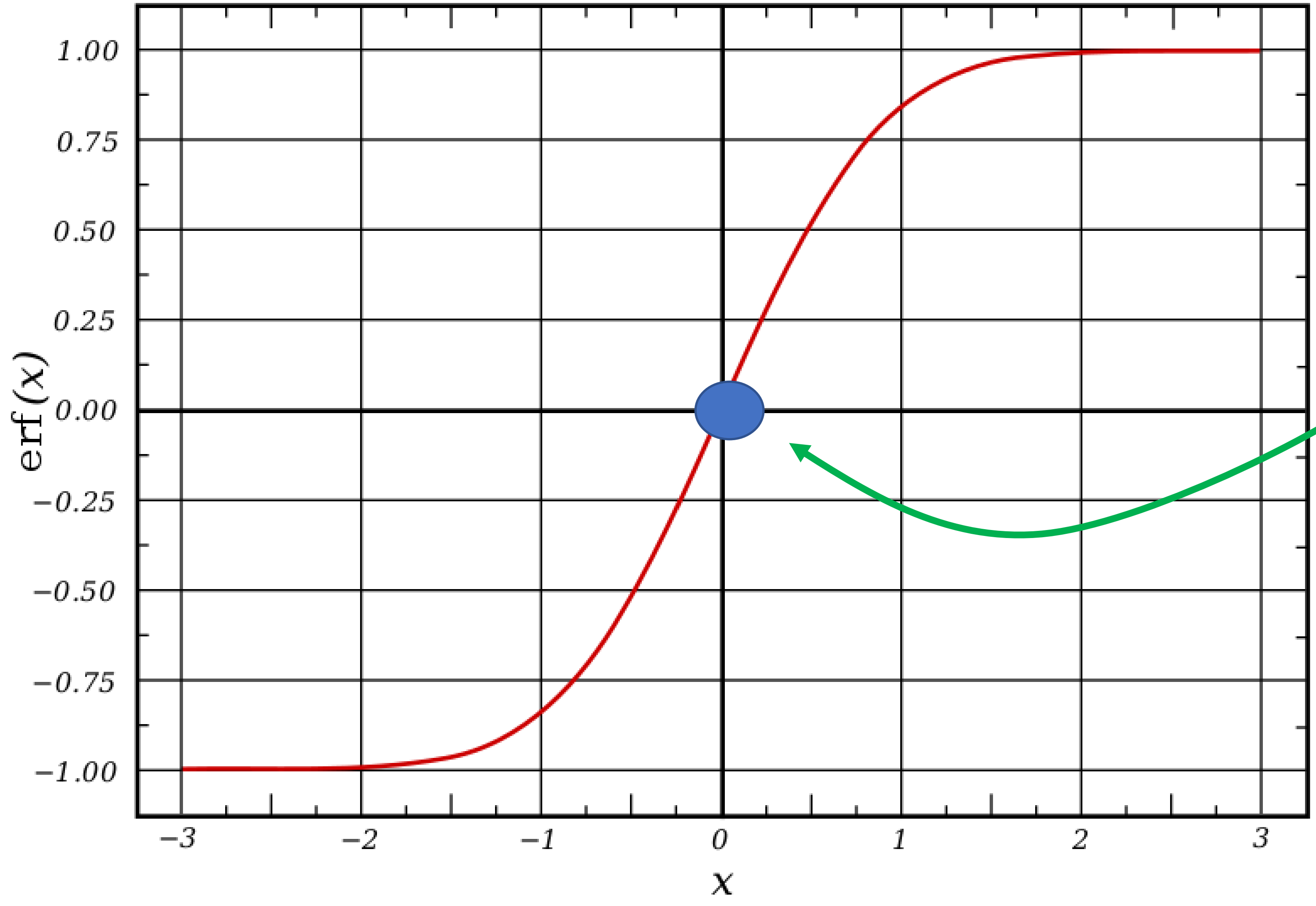
...

Zeta function

Error function, $\mathrm{erf}(x)$

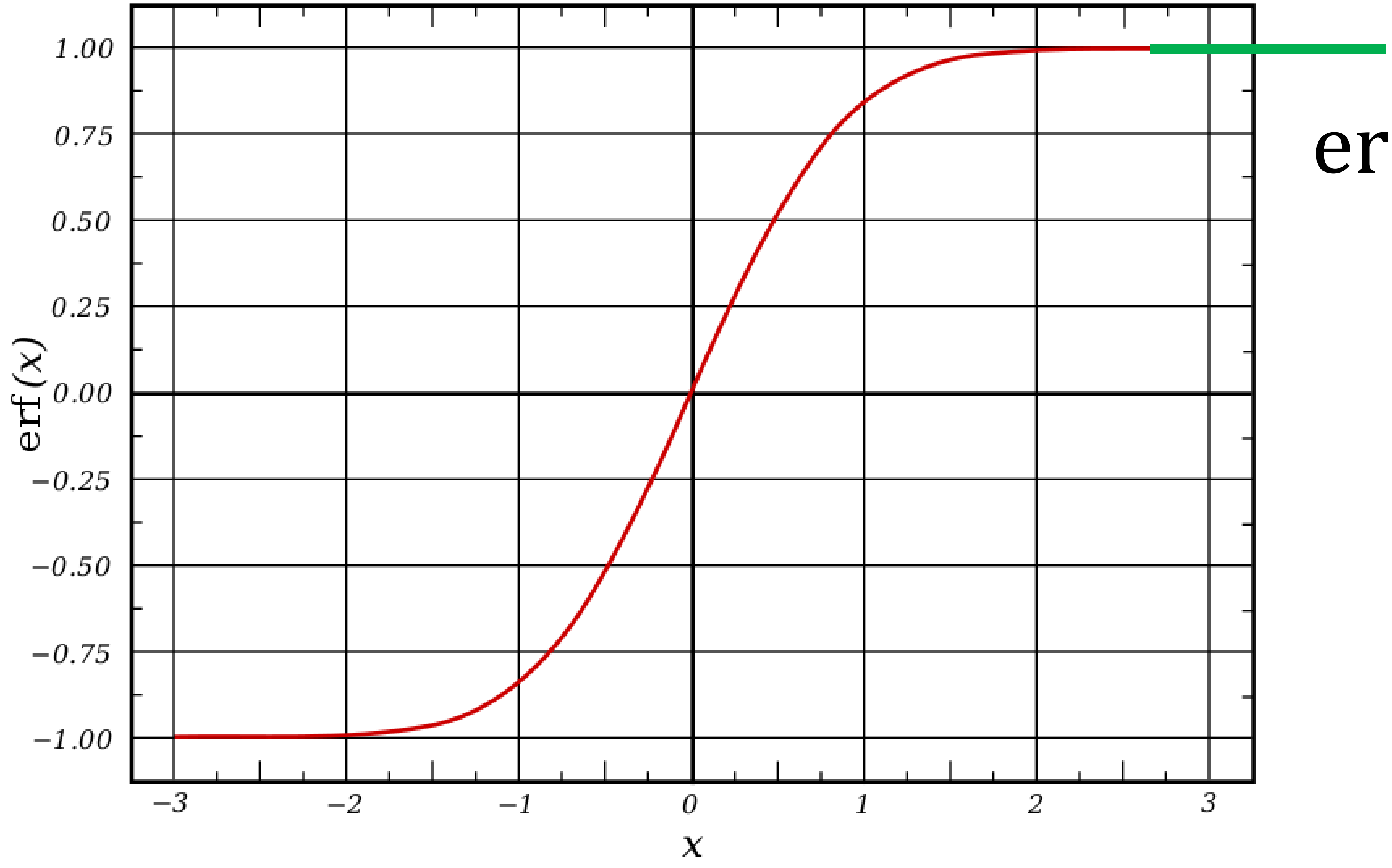


Error function, $\mathrm{erf}(x)$



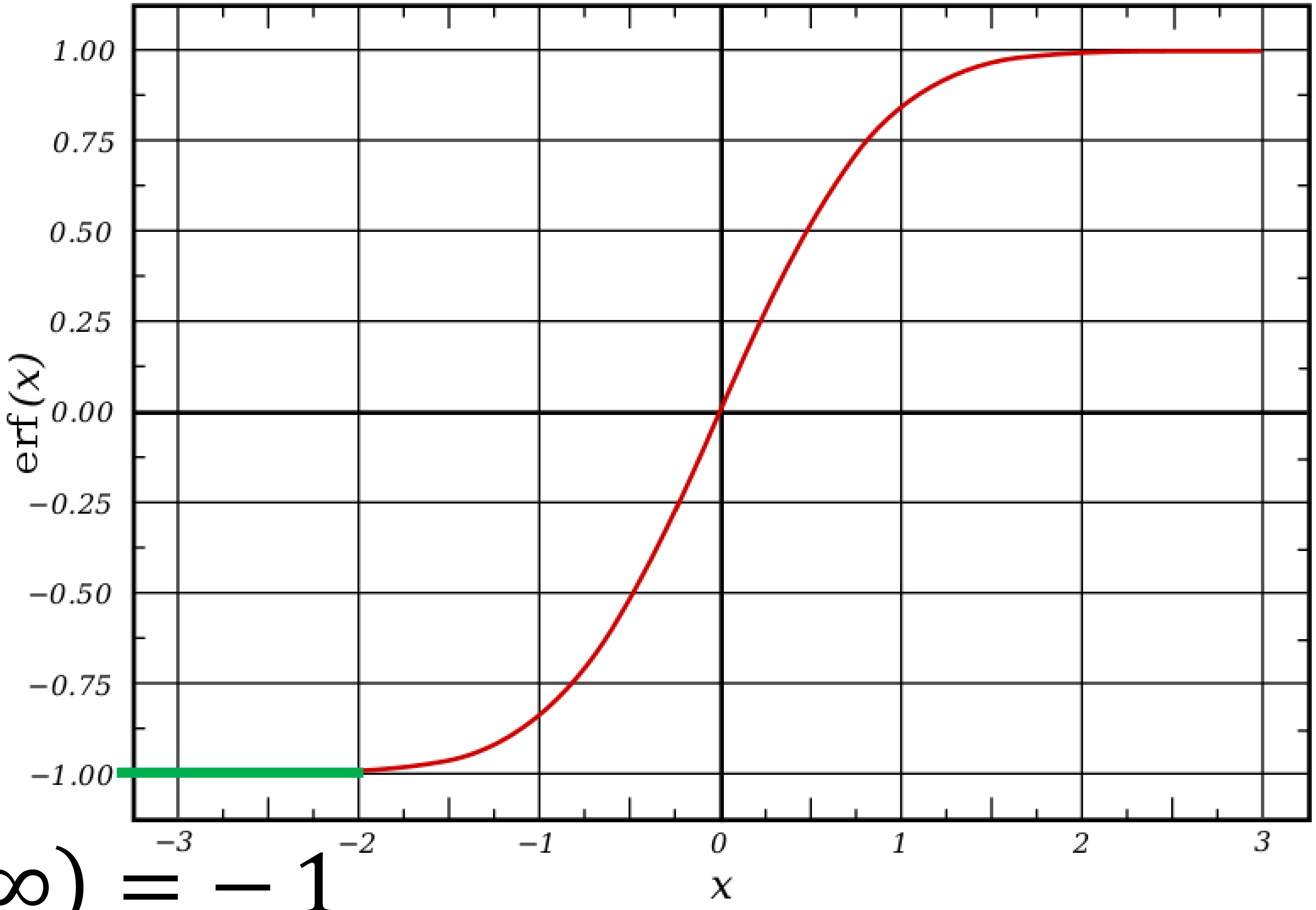
$\mathrm{erf}(0) = 0$

Error function, $\mathrm{erf}(x)$



$$\mathrm{erf}(\infty) = 1$$

Error function, $\mathrm{erf}(x)$

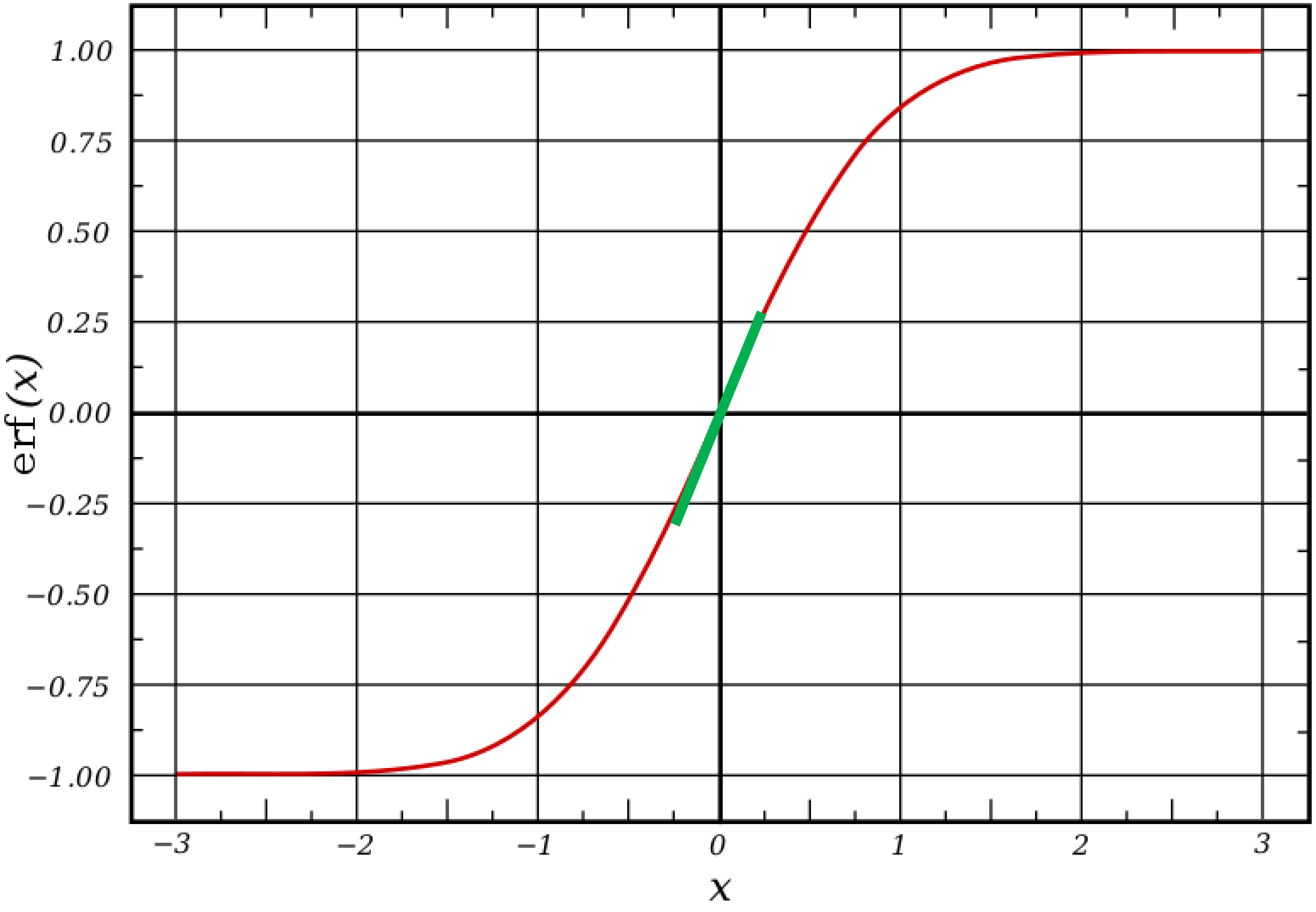


$$\mathrm{erf}(-\infty) = -1$$

Error function, $\mathrm{erf}(x)$

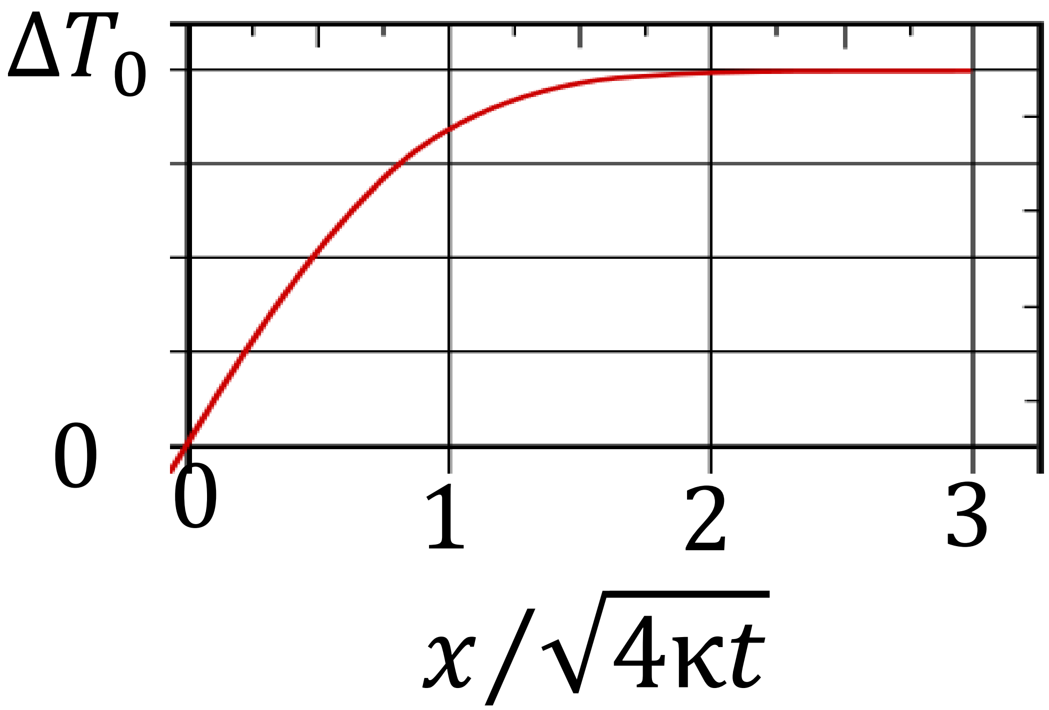
steepest near origin

slope $\frac{2}{\sqrt{\pi}}$

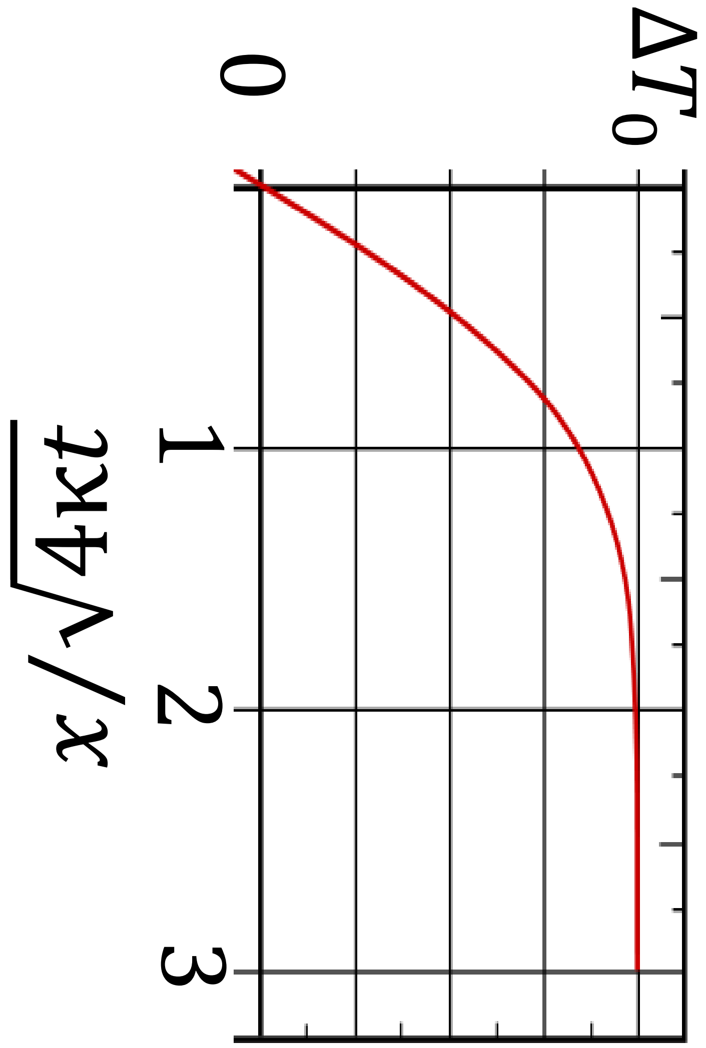


$$\Delta T(x,t) = \Delta T_0 \ \operatorname{erf}\left\{\frac{x}{\sqrt{4\kappa t}}\right\}$$

we only care about the positive-x part

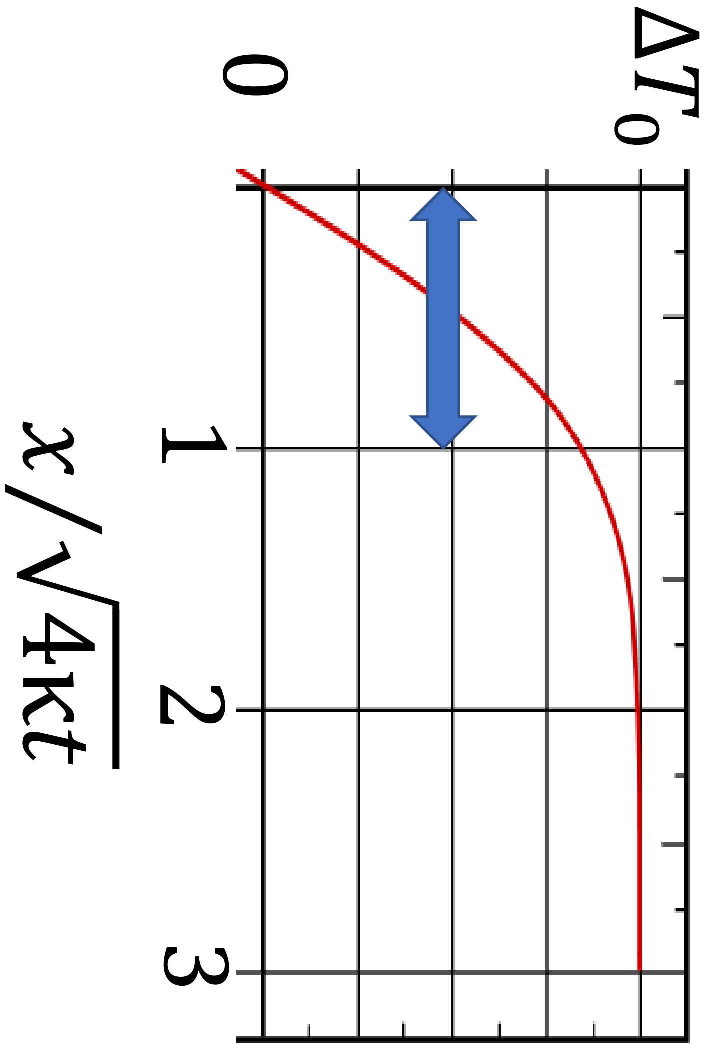


$$\Delta T(x,t) = \Delta T_0 \ \operatorname{erf}\left\{\frac{x}{\sqrt{4\kappa t}}\right\}$$



and we turn it sideways

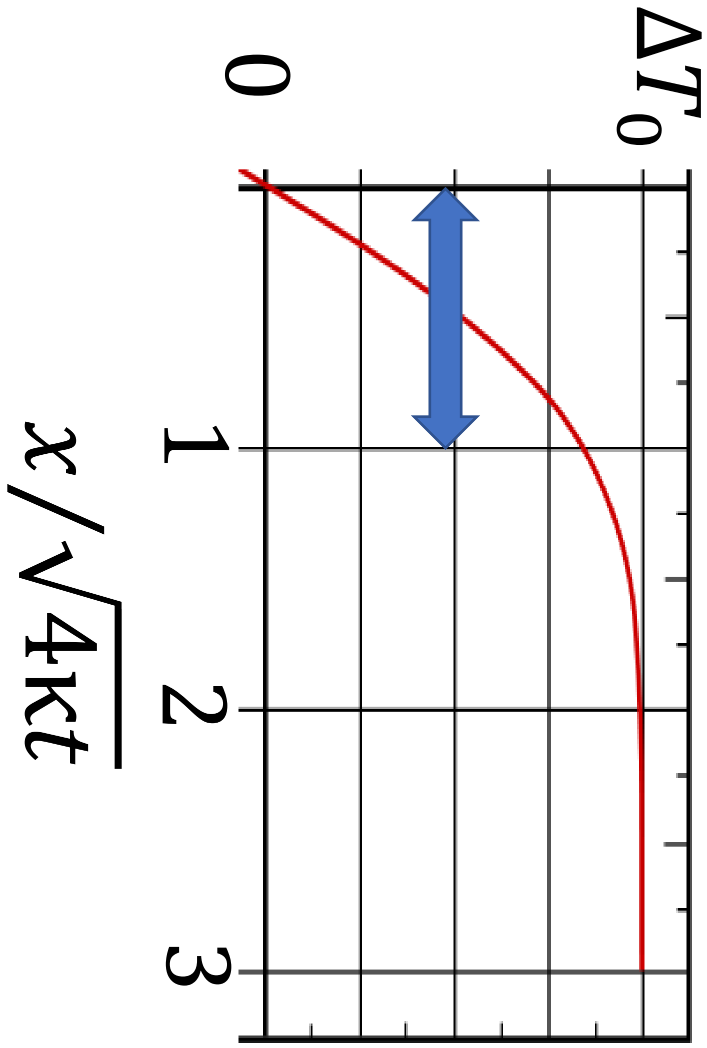
$$\Delta T(x,t) = \Delta T_0 \;\; \mathrm{erf}\left\{\frac{x}{\sqrt{4\kappa t}}\right\}$$



the “cooled part” with

$$0 \leq \frac{x}{\sqrt{4\kappa t}} \leq 1$$

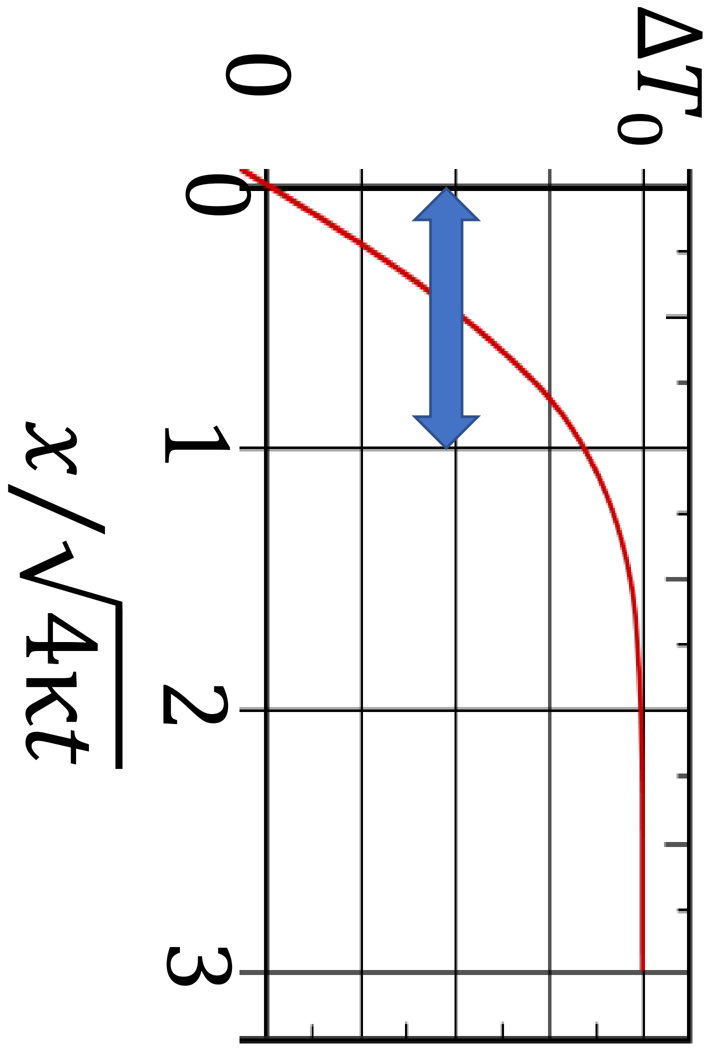
$$\Delta T(x,t) = \Delta T_0 \ \mathrm{erf}\left\{\frac{x}{\sqrt{4\kappa t}}\right\}$$



"bottom of
cooled part at"

$$x = \sqrt{4\kappa t}$$

$$\Delta T(x,t) = \Delta T_0 \ \mathrm{erf}\left\{\frac{x}{\sqrt{4\kappa t}}\right\}$$



“bottom of
cooled part at”

$$x = \sqrt{4\kappa t}$$

the bottom of the cool part deepens with the square root or time

$$x = \sqrt{4\kappa t} = \sqrt{\frac{4kt}{\rho c_p}}$$

granite

$$\rho = 2500\ \frac{kg}{m^3}$$

$$c_p = 720\,\frac{\mathrm{J}}{\mathrm{kg°C}}$$

$$k = 3.1\,\frac{J}{sm°\mathrm{C}}$$

how thick is the cooled zone after 100 million years?

$$1\ \mathrm{yr}\ = 3.1 \times 10^{7}\ \mathrm{s}$$

$$10^{8} yr\ = 3.1 \times 10^{15}\ \mathrm{s}$$

$$x = \sqrt{\frac{4kt}{\rho c_p}} \qquad k = 3.1\,\frac{J}{sm°\mathrm{C}} \qquad c_p = 720\,\frac{\mathrm{J}}{\mathrm{kg°C}}$$

$$\rho = 2500\,\frac{kg}{m^3} \qquad t = 3.1 \times 10^{15}\ \mathrm{s}$$

$$x = \sqrt{\frac{4 \times 3.1 \times 3.1 \times 10^{15}}{2500 \times 720}\,\frac{\mathrm{J} \times \mathrm{m}^3 \times \mathrm{s} \times \mathrm{kg°C}}{\mathrm{sm°C} \times \mathrm{kg} \times \mathrm{J}}}$$

$$x = \sqrt{2.14 \times 10^{10}\mathrm{m}^2} = 146000\ \mathrm{m} = 146\ \mathrm{km}$$

$$x = \sqrt{\frac{4kt}{\rho c_p}}$$

$$k = 3.1 \frac{J}{sm°C} \qquad c_p = 720 \frac{\mathrm{J}}{\mathrm{kg°C}}$$

$$\rho = 2500 \frac{kg}{m^3} \qquad t = 3.1 \times 10^{15}\ \mathrm{s}$$

$$x = \sqrt{\frac{4 \times 3.1 \times 3.1 \times 10^{15}}{2500 \times 720} \frac{\mathrm{J} \times \mathrm{m}^3 \times \mathrm{s} \times \mathrm{kg°C}}{\mathrm{sm°C} \times \mathrm{kg} \times \mathrm{J}}}$$

$$x = \sqrt{2.14 \times 10^{10}\mathrm{m}^2} = 146000\ \mathrm{m} = 146\ \mathrm{km}$$

roughly the same thickness as the lithosphere

$$x = \sqrt{\frac{4kt}{\rho c_p}} \qquad k = 3.1\,\frac{J}{sm°C} \qquad c_p = 720\,\frac{\mathrm{J}}{\mathrm{kg°C}}$$

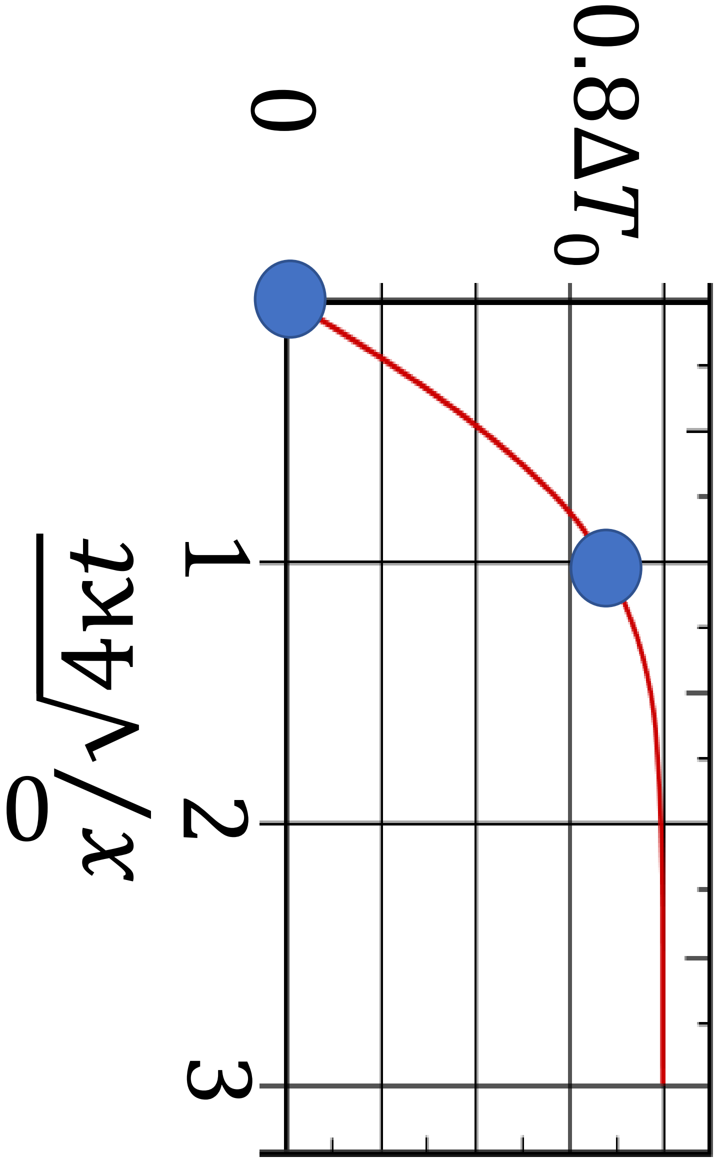
$$\rho = 2500\,\frac{kg}{m^3} \qquad t = 3.1 \times 10^{15}\ \mathrm{s}$$

$$x = \sqrt{\frac{4 \times 3.1 \times 3.1 \times 10^{15}}{2500 \times 720}\,\frac{\mathrm{J} \times \mathrm{m}^3 \times \mathrm{s} \times \mathrm{kg°C}}{\mathrm{sm°C} \times \mathrm{kg} \times \mathrm{J}}}$$

$$x = \sqrt{2.14 \times 10^{10}\mathrm{m}^2} = 146000\ \mathrm{m} = 146\ \mathrm{km}$$

maybe that's not a coincidence!

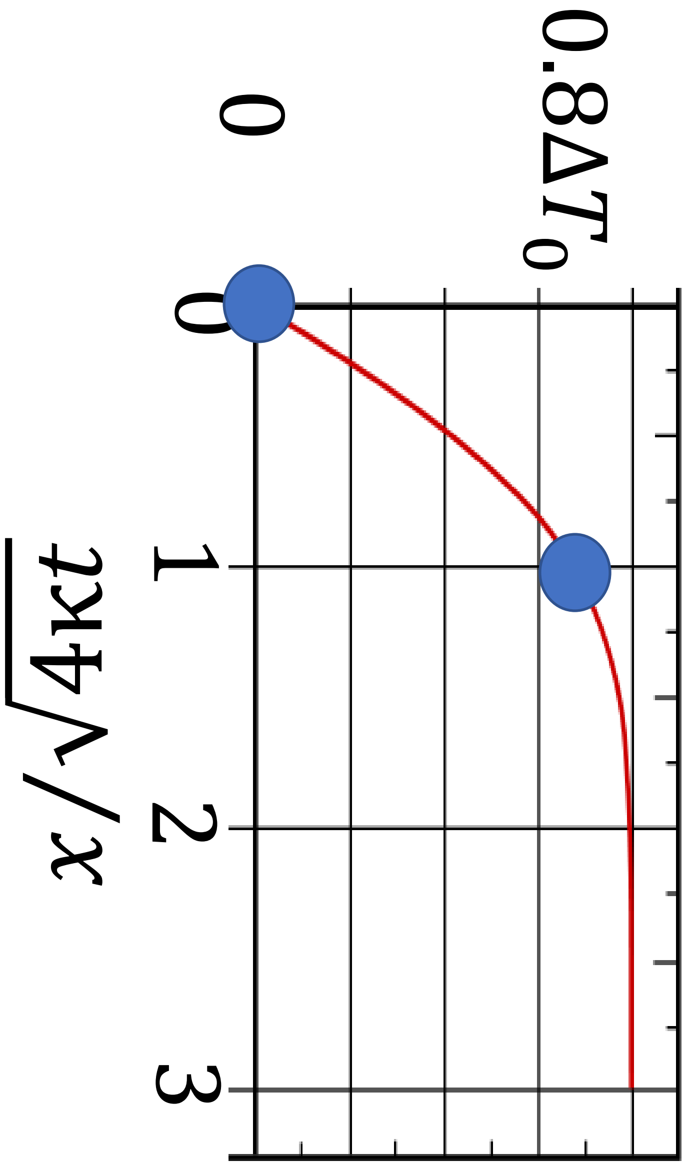
$$\Delta T(x,t) = \Delta T_0 \ \operatorname{erf}\left\{\frac{x}{\sqrt{4\kappa t}}\right\}$$



what's the average temperature of the "cooled part"?

bottom 0
top about $0.8\Delta T_0$
pretty linear in between

$$\Delta T(x,t) = \Delta T_0 \ \ \text{erf}\left\{\frac{x}{\sqrt{4\kappa t}}\right\}$$



what's the average temperature of the "cooled part"?

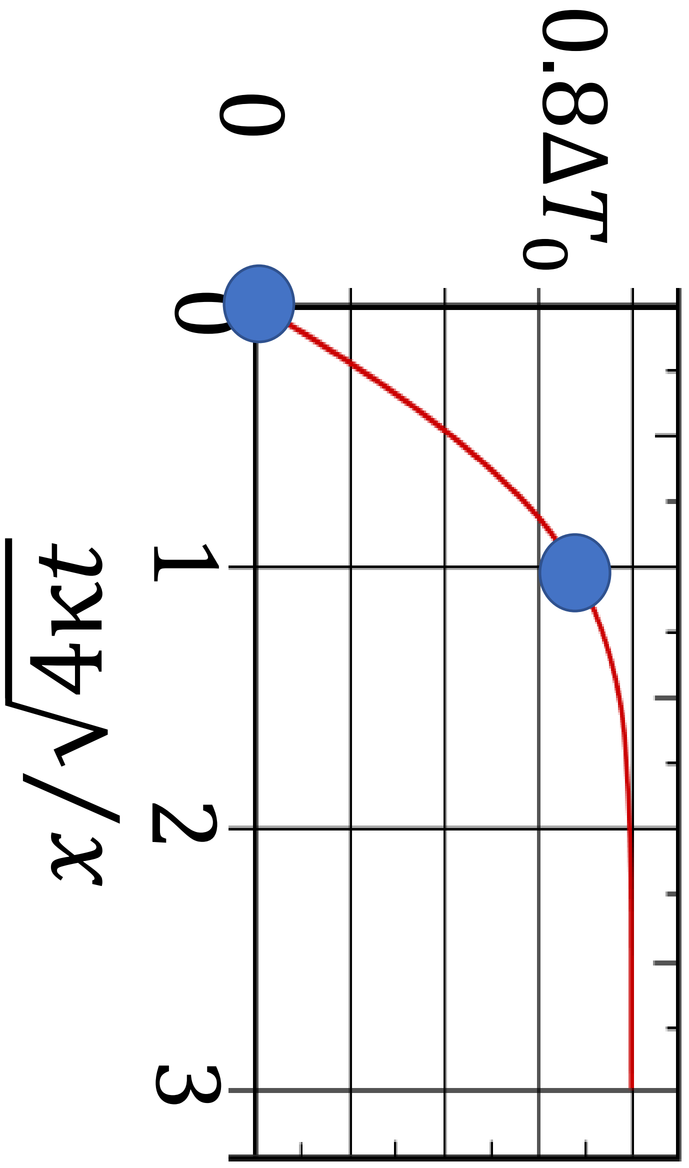
so about

top about $0.4\Delta T_0$

$$\Delta T(x,t) = \Delta T_0 \ \mathrm{erf}\left\{\frac{x}{\sqrt{4\kappa t}}\right\}$$

How much did it cool?

started at ΔT_0
cooled on average to
about $0.4\Delta T_0$

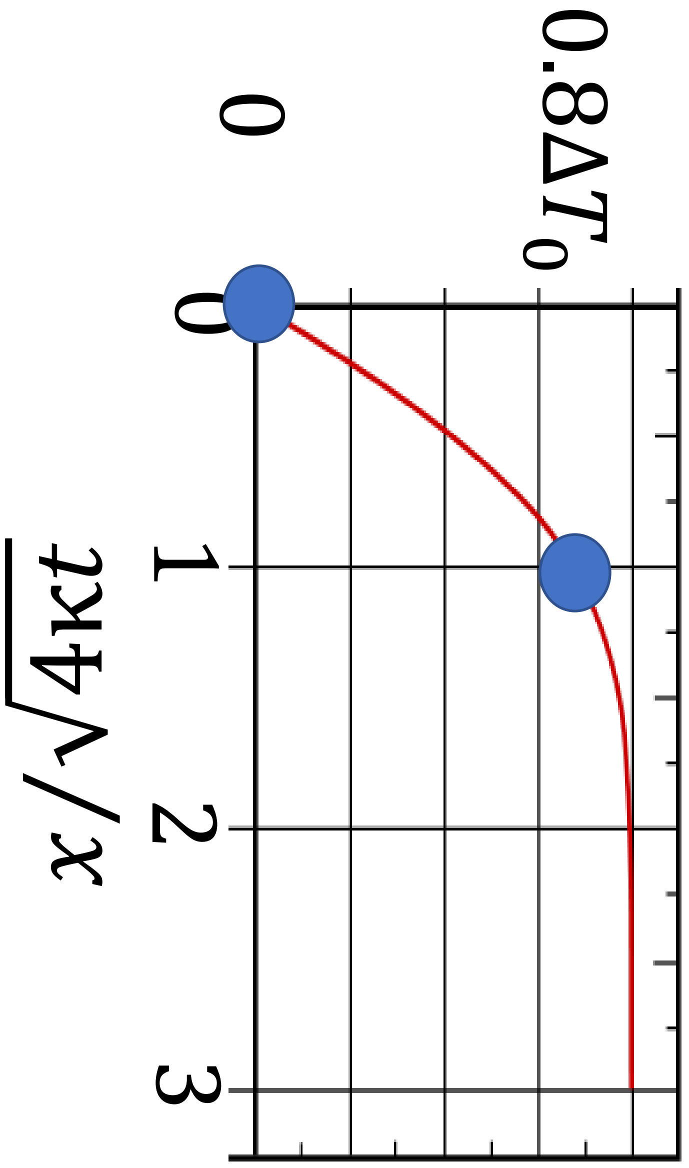


$$\Delta T(x,t) = \Delta T_0 \ \mathrm{erf}\left\{\frac{x}{\sqrt{4\kappa t}}\right\}$$

How much did it cool?

started at ΔT_0
cooled on average to
about $0.4\Delta T_0$

so cooled $0.6\Delta T_0$



Thermal expansion and contraction

fractional change in length L of a material is proporpional to the chance in temperature

$$\frac{\Delta L}{L} = \alpha \Delta T$$

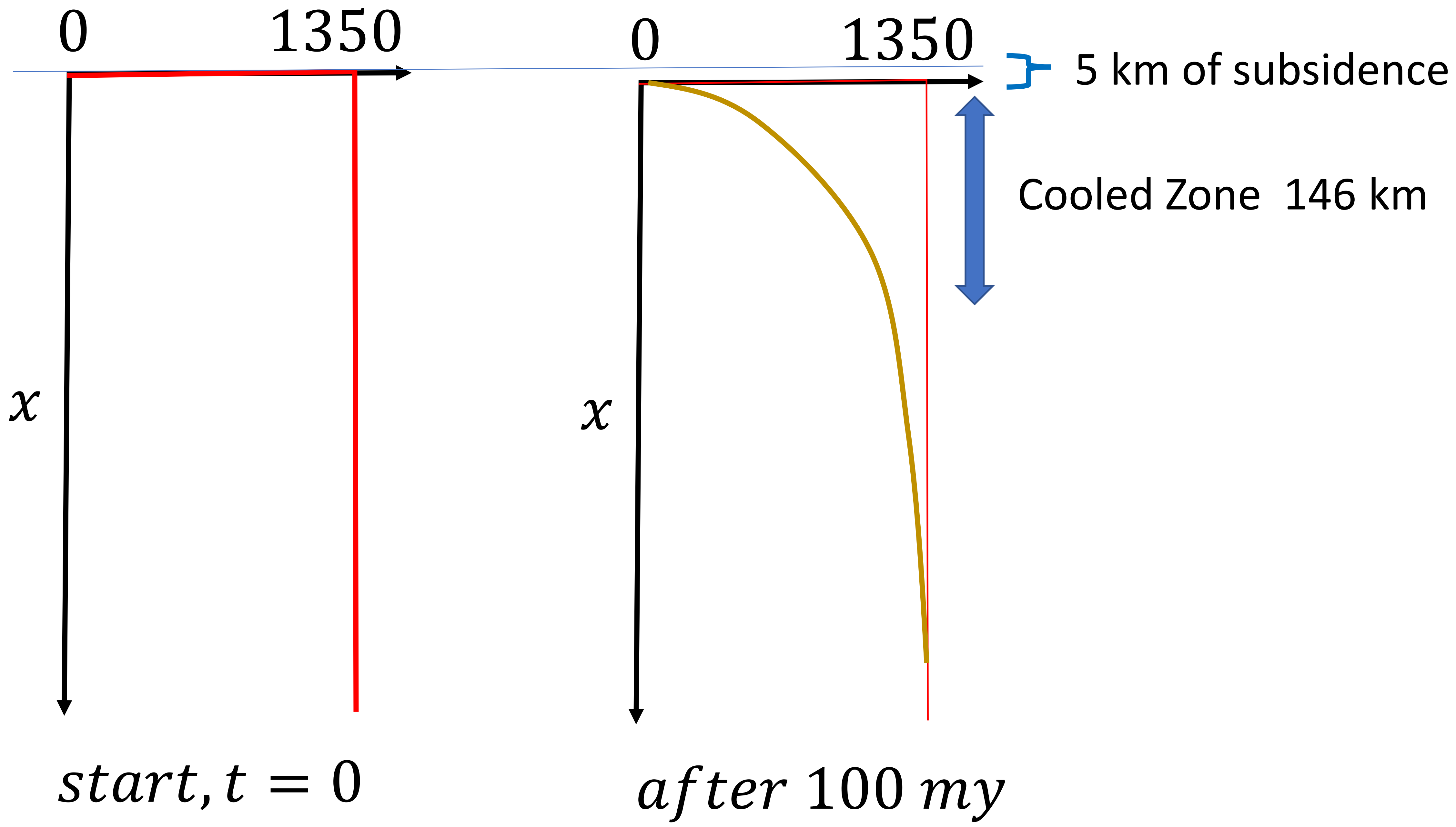
coefficient of thermal expansion α

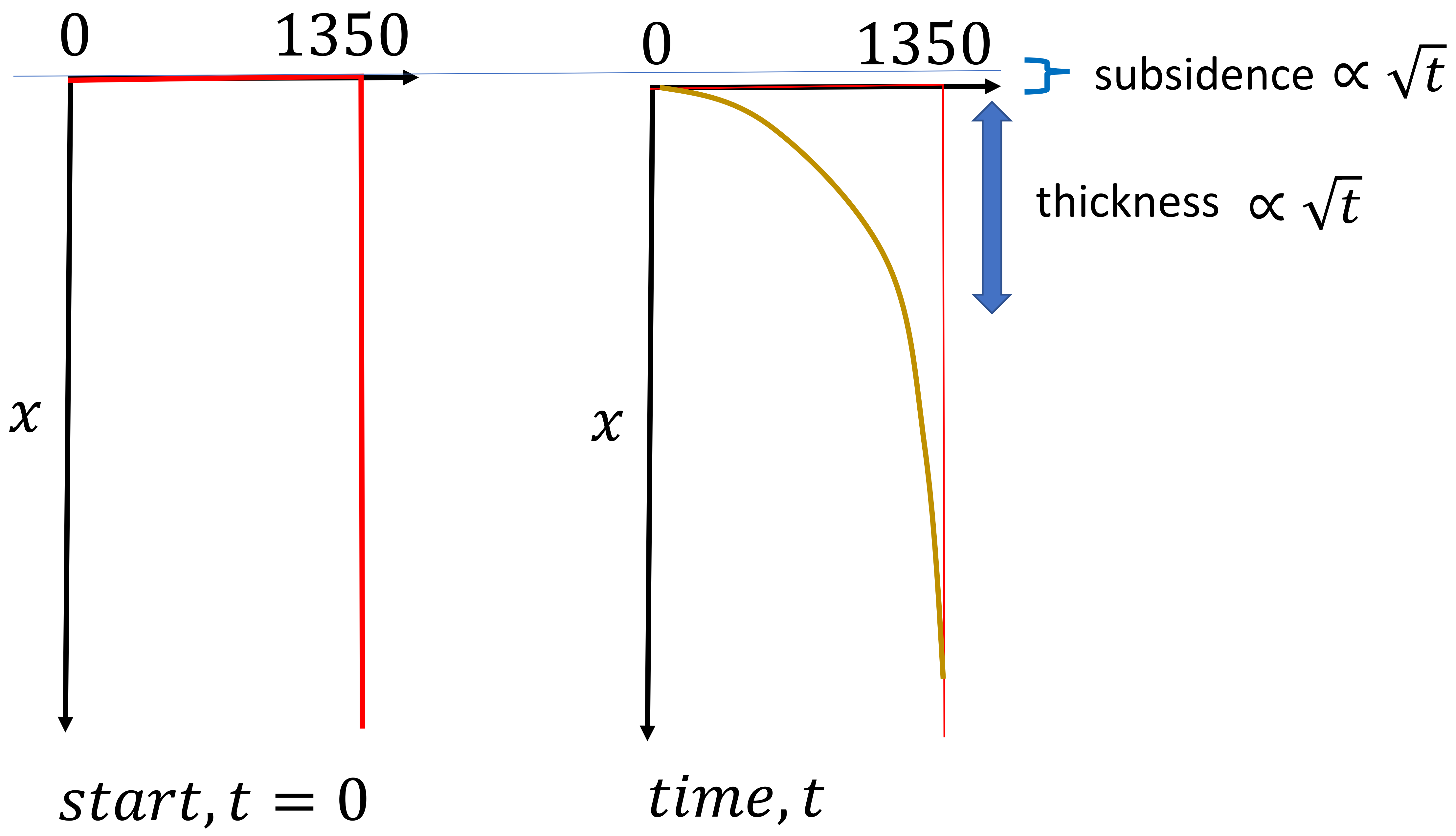
for granite $\alpha = 4 \times 10^{-5}\ \frac{1}{°\mathrm{C}}$

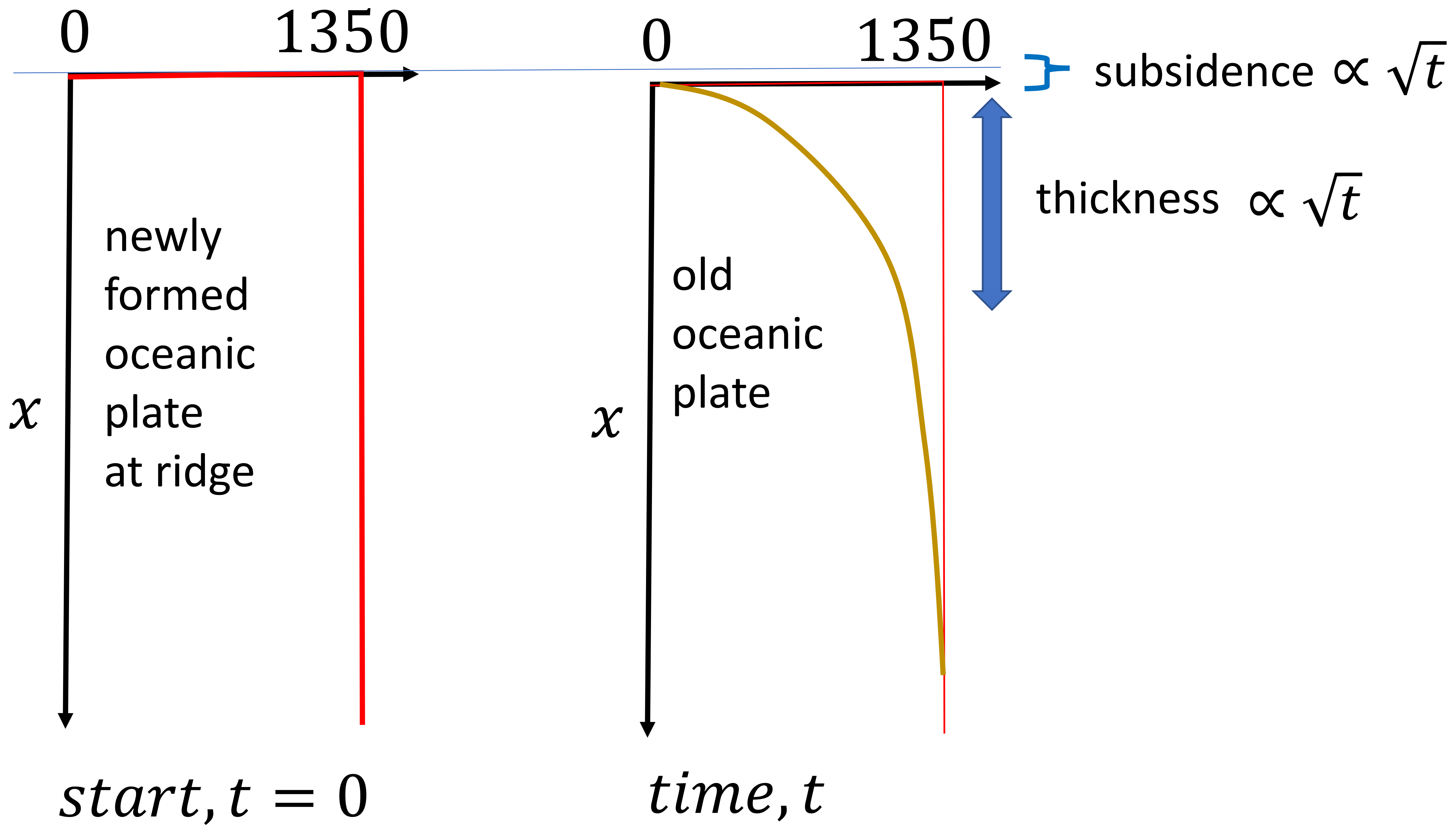
starting at $\Delta T_0 = 1350\ ℃$
after 100 million years

thickness of the cooled part $L = 146000\ \mathrm{m}$
change in temperature $0.6\Delta T_0 = 810\ ℃$

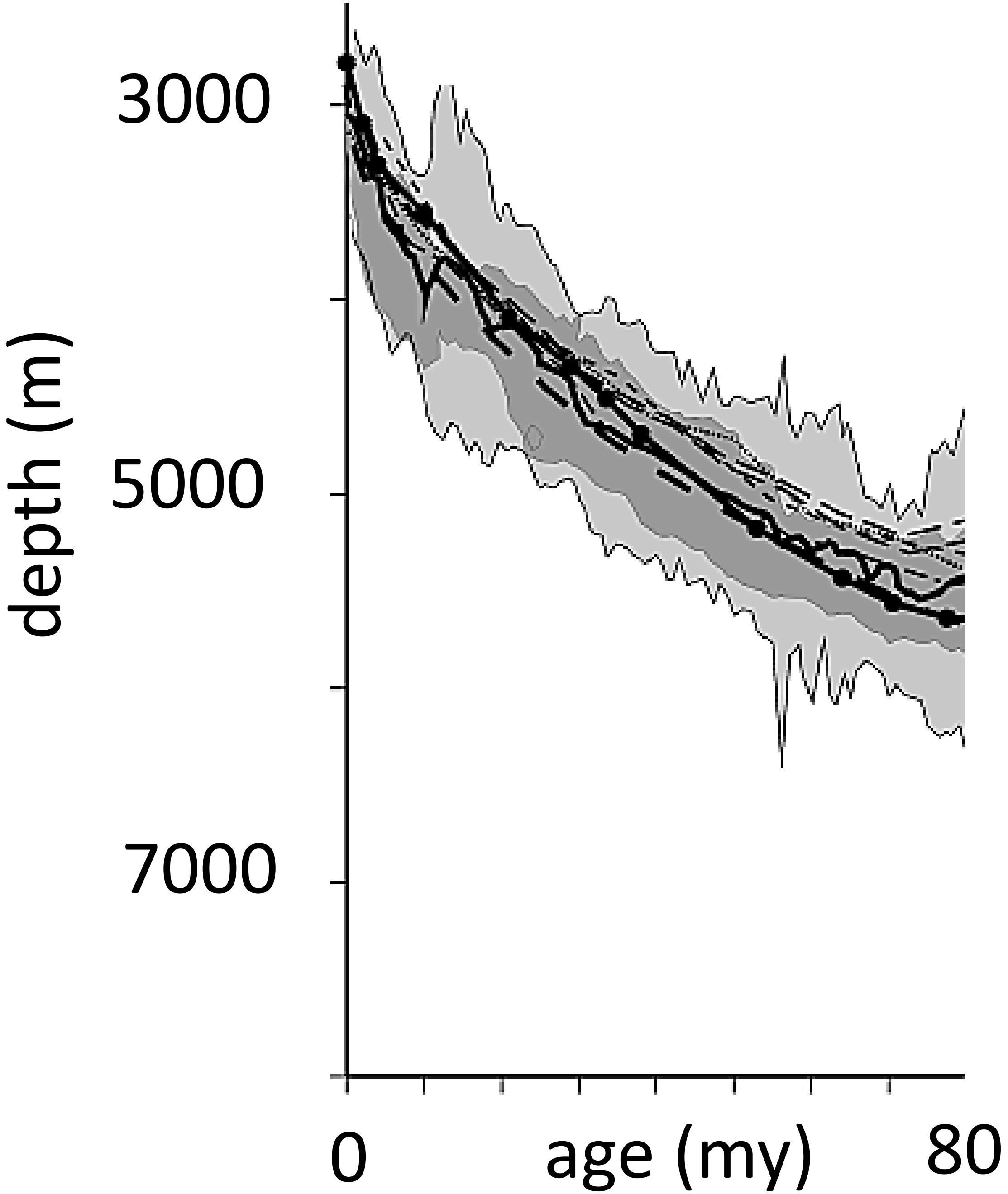
$$\frac{\Delta L}{L} = \alpha \Delta T \quad \text{so} \quad \Delta L = \alpha \Delta T L = 4730\ \mathrm{m}$$


0
1350
x
start, t = 0
0
1350
5 km of subsidence
Cooled Zone 146 km
x
after 100 my

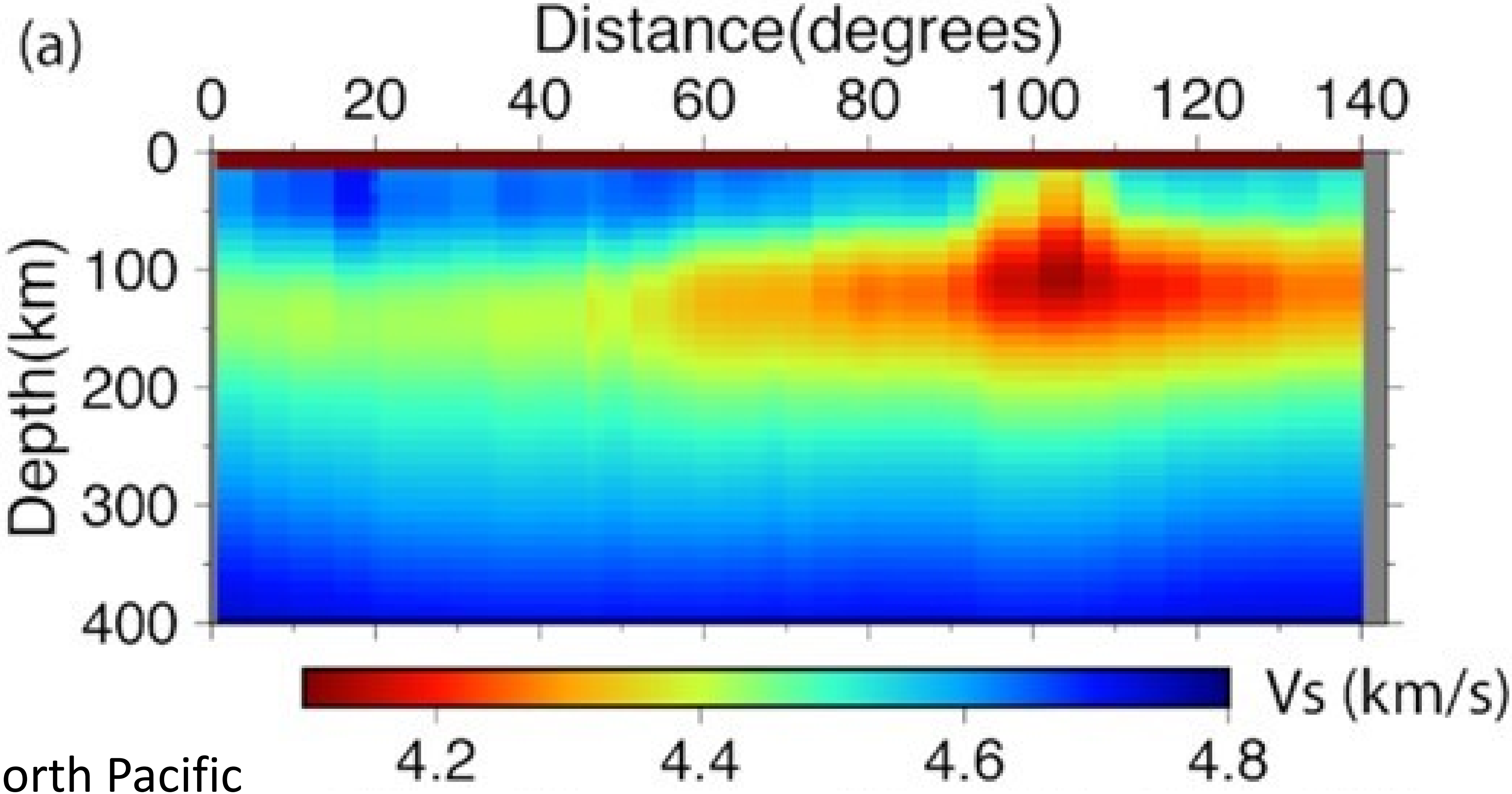

0
1350
x
$start, t = 0$
0
1350
subsidence $\propto \sqrt{t}$
thickness $\propto \sqrt{t}$
x
$time, t$


0
1350
newly
formed
oceanic
plate
at ridge
x
start, t = 0
0
1350
subsidence ∝ √t
thickness ∝ √t
old
oceanic
plate
x
time, t

North Pacific



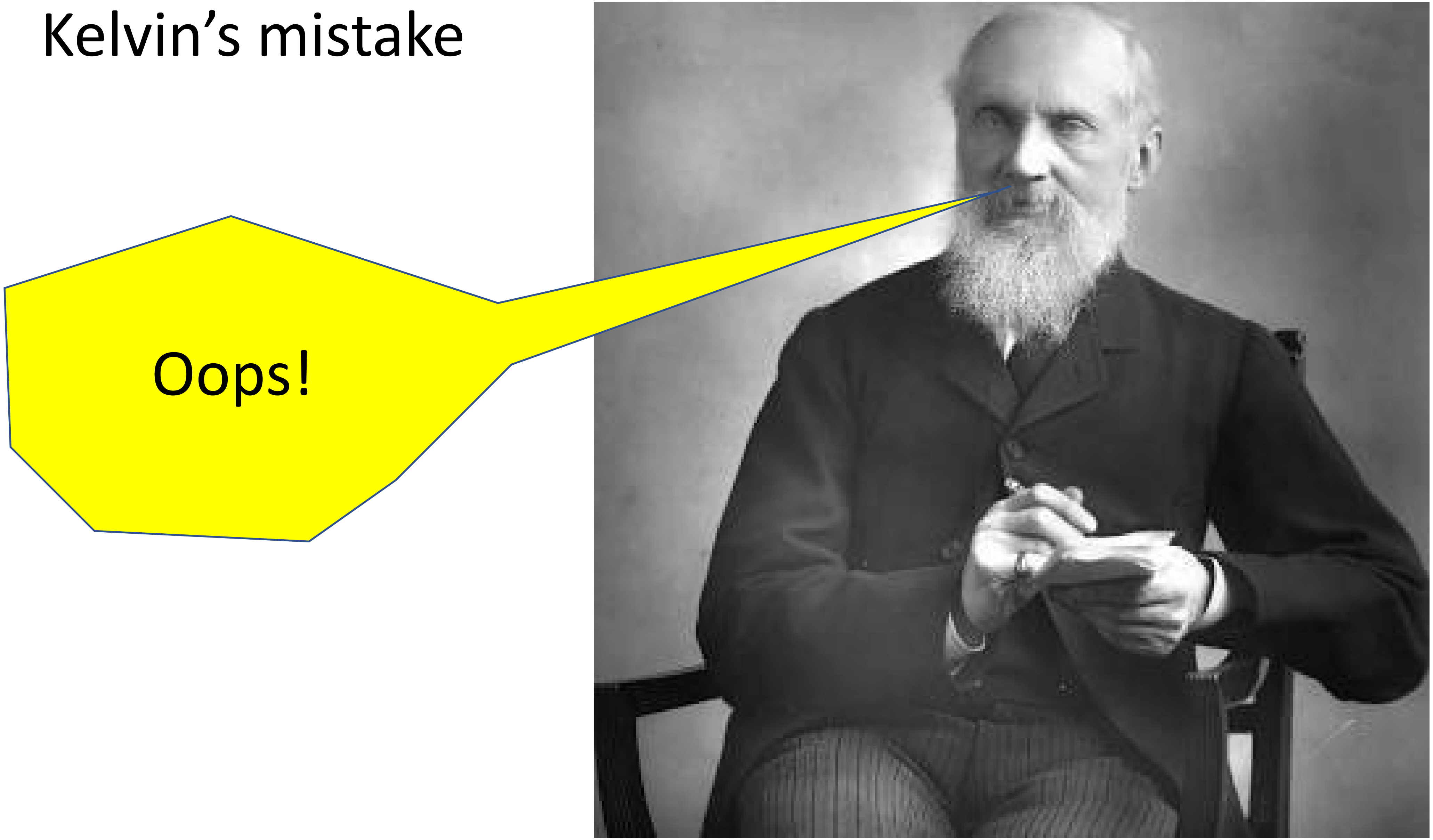
J. K. Hillier, A. B. Watts, 2005



North Pacific

Celia L Eddy, Göran Ekström, Meredith Nettles, 2022

Kelvin’s mistake



Kelvin's mistake

temperature

$$\Delta T(x,t) = \Delta T_0 \ \ \mathrm{erf}\left\{\frac{x}{\sqrt{4\kappa t}}\right\}$$

spatial derivative

$$\frac{d}{dx}\Delta T(x=0) = \frac{\Delta T_0}{\sqrt{4\kappa t}}\left[\frac{d}{dx}\mathrm{erf}\{x\}\right]_{x=0} = \frac{\Delta T_0}{\sqrt{4\kappa t}}\frac{2}{\sqrt{\pi}}$$

surface heat flow

$$q = -k\frac{d\Delta T}{dx} = -2\frac{\Delta T_0 k}{\sqrt{4\pi\kappa t}}$$

Kelvin’s mistake

surface heat flow $q = -2\frac{\Delta T_0 k}{\sqrt{4\pi\kappa t}}$

at what time is hear flow $q = 0.06\ \mathrm{W/m^2}$?

$$t = \frac{(\Delta T_0)^2 k\rho c_p}{\pi q^2} = 9 \times 10^{14} s$$

$$= 30 \text{ million years}$$

Kelvin’s mistake

surface heat flow $q = -2\frac{\Delta T_0 k}{\sqrt{4\pi\kappa t}}$

at what time is hear flow $q = 0.06\ \mathrm{W/m^2}$?

$$t = \frac{(\Delta T_0)^2 k\rho c_p}{\pi q^2} = 9 \times 10^{14} s$$

$$= 30\ \text{million years}$$

Kelvin’s mistake was to interpret this number of the age of the Earth

Kelvin's mistake

surface heat flow $q = -2\dfrac{\Delta T_0 k}{\sqrt{4\pi\kappa t}}$

at what time is hear flow $q = 0.06\ \mathrm{W/m^2}$?

$$t = \frac{(\Delta T_0)^2 k\rho c_p}{\pi q^2} = 9 \times 10^{14} s$$

$= 30$ million years

What is really is the age the ocean plate that has cooled enough for its heat flow to drop to 0.06 W/m^2