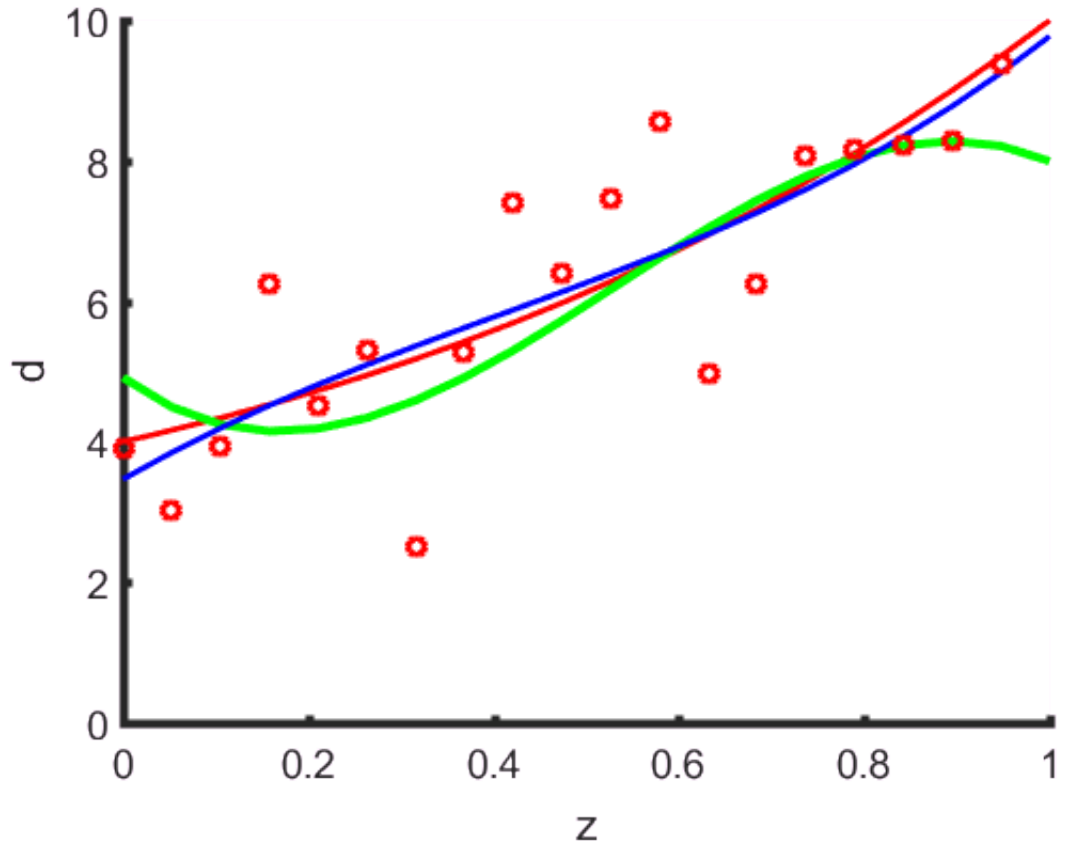
```
% gda08_05
%
% Linf norm inverse problem, overdetermined case
% Supports Figure 8.5

% auxillary variable, z
N=20;
zmin = 0;
zmax = 1;
Dz = (zmax-zmin)/(N-1);
z = zmin + Dz*[0:N-1]';

% set up for cubic fit
M=4;
mtrue = [4, 3, 2, 1]';
G = [ones(N,1), z, z.^2, z.^3];
dtrue = G*mtrue;
sd = 1 * ones(N,1);
dobs=zeros(N,1);

for i=[1:N]
    r=random('exp',sd(i)/sqrt(2)).*(2*(random('unid',2)-1.5));
    dobs(i) = dtrue(i) + r;
end

% outlier
dobs( floor(0.37*N) ) = 2.5;

% least squares solution (sure eassier!)
mls = (G'*G)\(G'*dobs);
dls = G*mls;

% Linf solution
% set up for linear programming problem
% min f*x subject to A x <= b,  Aeq x = beq

% variables
% m = mp - mpp
% x = [mp', mpp', alpha', x', xp']
% with mp, mpp length M; alpha length 1, x, xp, length N
L = 2*M+1+2*N;
x = zeros(L,1);

% f is length L
% minimize alpha
f = zeros(L,1);
f(2*M+1:2*M+1)=1;

% equality constraints
Aeq = zeros(2*N,L);
beq = zeros(2*N,1);
% first equation G(mp-mpp)+x-alpha=d
Aeq(1:N,1:M)             =  G;
Aeq(1:N,M+1:2*M)         = -G;
Aeq(1:N,2*M+1:2*M+1)     = -1./sd;
Aeq(1:N,2*M+1+1:2*M+1+N) =  eye(N,N);
beq(1:N)                 =  dobs;
% second equation G(mp-mpp)-xp+alpha=d
Aeq(N+1:2*N,1:M)                 =  G;
```

```
Aeq(N+1:2*N,M+1:2*M)             = -G;
Aeq(N+1:2*N,2*M+1:2*M+1)         =  1./sd;
Aeq(N+1:2*N,2*M+1+N+1:2*M+1+2*N) = -eye(N,N);
beq(N+1:2*N)                   =  dobs;

% inequality constraints A x <= b
% part 1: everything positive
A = zeros(L+2*M,L);
b = zeros(L+2*M,1);
A(1:L,:) = -eye(L,L);
b(1:L) = zeros(L,1);
% part 2; mp and mpp have an upper bound.  Note:
% Without this constraint, a potential problem is that
% mp and mpp are individually unbounded, though their
% difference, m=mp-mpp, is not.  Thus there might be cases
% where the algroithm strays to very large mp and mpp.
A(L+1:L+2*M,:) = eye(2*M,L);
mupperbound=10*max(abs(mls)); % might need to be adjusted for problem at hand
b(L+1:L+2*M) = mupperbound;

% I need to add an 'options' argument to linprog() for
% MATLAB release 2013a and above, so determine the release year
nMLver = sscanf(version('-release'),'%d');
if( nMLver >= 2013 )
    options = optimoptions('linprog','Algorithm','interior-point-legacy');
end

% solve linear programming problem
if( nMLver>= 2013 )
    [x, fmin] = linprog(f,A,b,Aeq,beq,[],[],options);
else
    [x, fmin] = linprog(f,A,b,Aeq,beq);
end
```

```
Optimization terminated.
```

```
fmin=-fmin;
mest = x(1:M) - x(M+1:2*M);
dpre = G*mest;

% plot
figure(1);
clf;
set(gca,'LineWidth',3);
set(gca,'FontSize',14);
hold on;
axis( [zmin, zmax, 0, max(dtrue) ] );
plot( z, dtrue, 'r-', 'Linewidth', 2);
plot( z, dpre, 'g-', 'Linewidth', 3);
plot( z, dls, 'b-', 'Linewidth', 2);
plot( z, dobs, 'ro', 'Linewidth', 2);
xlabel('z');
ylabel('d');
```




```
% Figure [AU Note: was 8.4] 8.5 Curve fitting using the L∞ norm. The true data (red curve) fol
% a cubic polynomial in an auxiliary variable z. Observations (red circles) have additive nois
% zero mean and variance σ2 = 1 drawn from an exponential probability density function. Note t
% outlier at z ≈ 0.37. The L∞ fit (green curve) is more affected by the outlier than is a star
% L2 (least-squares) fit (blue curve). MatLab script [AU Note: ws gda08_04] gda08_05.
```