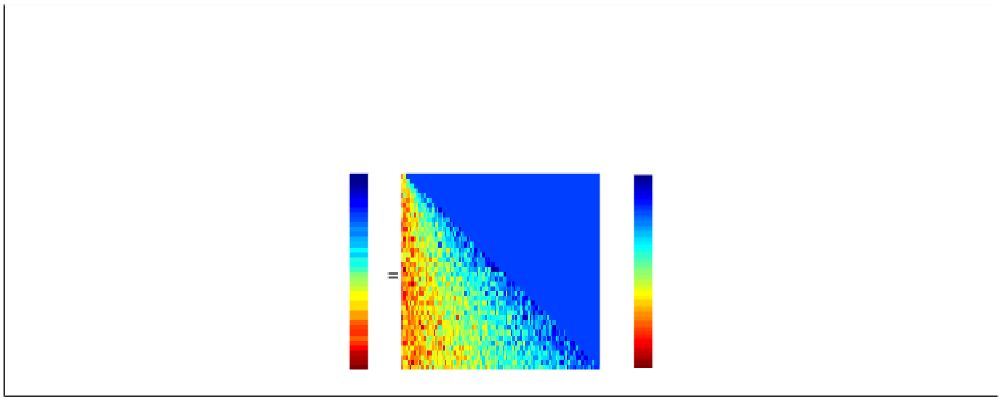
```
% gda06_02
%
% upper/lower bounds on localized average using linear programming
% supports Figure 6.2
% data kernel G composed of decaying exponentials

clear all;

% model, m(z), moztly zero but a few spikes
M=101;
zmin=0;
zmax=10;
Dz=(zmax-zmin)/(M-1);
z=zmin+Dz*[0:M-1]';
mtrue = 0.5+0.03*z;

% experiment: exponential smoothing of model
N=40;
G = zeros(N,M);
for i = 1:N;
    j = floor(i*M/N );
    % G(i,1:j) = [j:-1:1]/j;
    G(i,1:j)=[j:-1:1]/j+random('Normal',0,0.15,1,j);
end

fprintf('det %f\n',det(G*G'));
```

```
det 0.007660
```

```

% observed data is true data plus noise
sd=0.001;
dtrue = G*mtrue;
dobs = dtrue + random('Normal',0,sd,N,1);

% draw the data kernel
gda_draw(dtrue,'=',G,' ',mtrue);
```




```
% Figure 6.2 (A) This underdetermined inverse problem, d = Gm, has M = 100 model parameters mi
% and N = 40 data di. The data are weighted averages of the model parameters, from the surface
% down to a depth, z, that increases with index, i. The observed data  include additive noise.
```

```
% minimum length solution
epsilon=1e-12;
GMG = G'/(G*G'+epsilon*eye(N,N));
mest = GMG * dobs;
Rres = GMG*G;

% plot
figure(2);
clf;

% plot scale
pmmin=-0.5;
pmmax=1.5;

% plto the true model and the minimum length estimate
set(gca,'LineWidth',3);
set(gca,'FontSize',14);
hold on;
axis( [zmin, zmax, pmmin, pmmax ]' );
plot( z, mtrue, 'r-', 'LineWidth', 2);
plot( z, mest,  'b-', 'LineWidth', 2);
xlabel('z');
ylabel('m');

% upper bound: mi <= 1
mub = ones(M,1);

% lower bound: mi >= -1
mlb = -ones(M,1);

% half width of averaging kernel
hw = 10;

% I need to add an 'options' argument to linprog() for
% MATLAB release 2013a and above, so determine the release year
nMLver = sscanf(version('-release'),'%d');
if( nMLver >= 2013 )
    options = optimoptions('linprog','Algorithm','interior-point-legacy');
end


% compute upper, lower bounds on average
amin=zeros(M,1);
amax=zeros(M,1);
for i=[1:M]
    % averageing vector centered at M of width w
    a = zeros(M,1);
    j=i-hw;
    if( j<1 )
        j=1;
    end
    k=i+hw;
    if(k>M)
        k=M;
    end
    a(j:k)=1/(k-j+1);
    if( nMLver>= 2013 )
            [mest1, amin(i)] = linprog(  a, [], [], G, dobs, mlb, mub, options );
            [mest2, amax(i)] = linprog( -a, [], [], G, dobs, mlb, mub, options );
    else
            [mest1, amin(i)] = linprog(  a, [], [], G, dobs, mlb, mub );
```

```
            [mest2, amax(i)] = linprog( -a, [], [], G, dobs, mlb, mub );
            end
    amax(i)=-amax(i);
end
```

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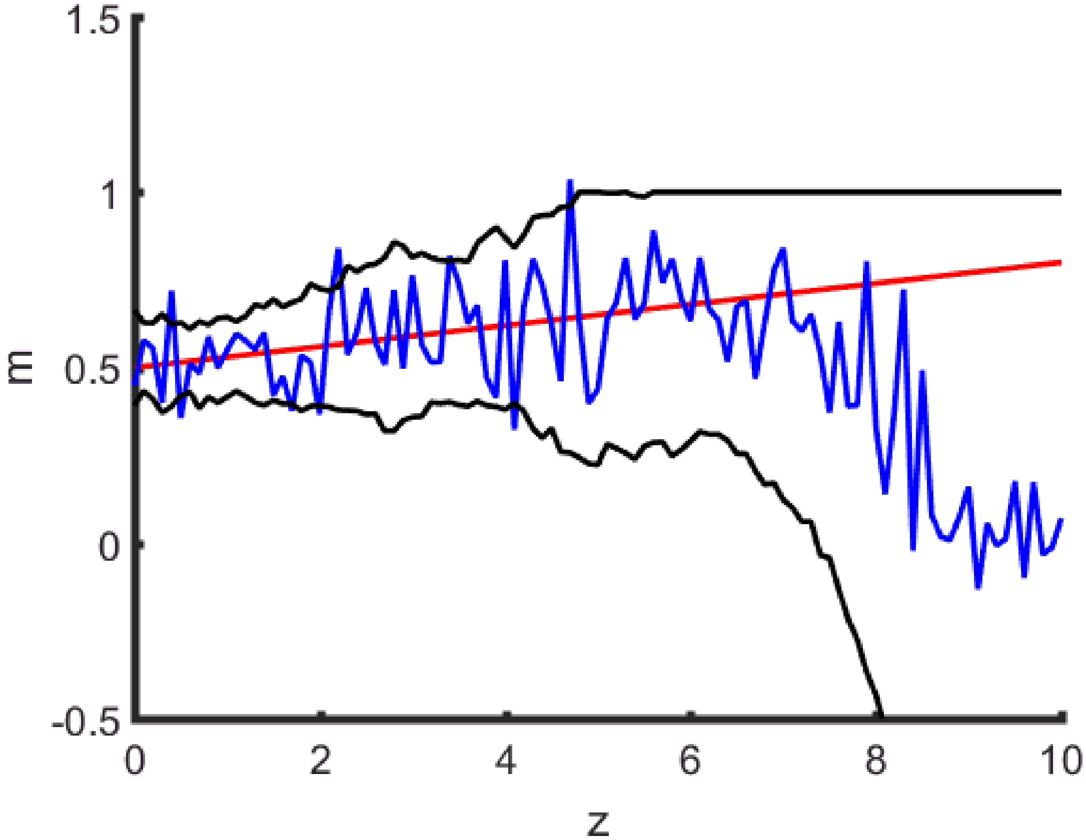
```
% plot bounds
plot( z, amin, 'k-', 'LineWidth', 2);
plot( z, amax, 'k-', 'LineWidth', 2);
```




```
% Figure 6.2 (B) The true model parameters (red curve) increase linearly with depth z. The est
% parameters (blue curve), computed using the minimum length method, scatter about the true mo
% at shallow depths (z < 6) but decline toward zero at deeper depths due to poor resolution.
% Bounds on localized averages of the model parameters, with an averaging width, w = 2 (black
% curves), are for [AU Note: replaced “a priori”] prior information, 0 < mi < 1 (gray dotted l
```