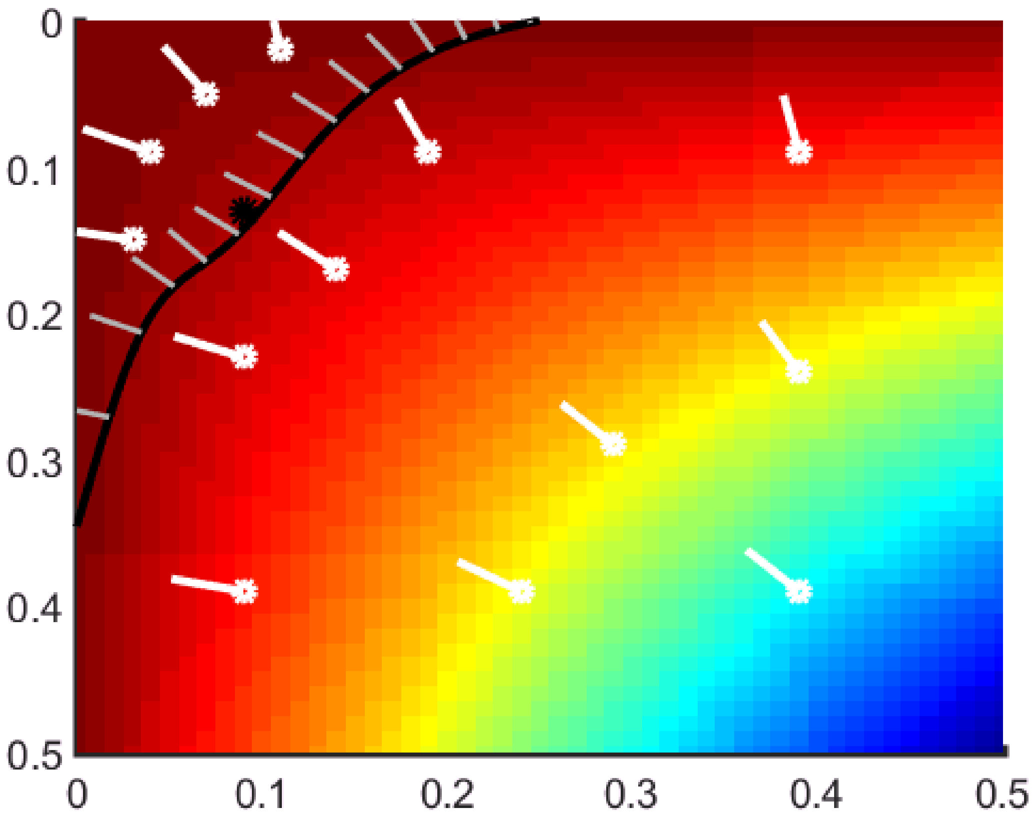
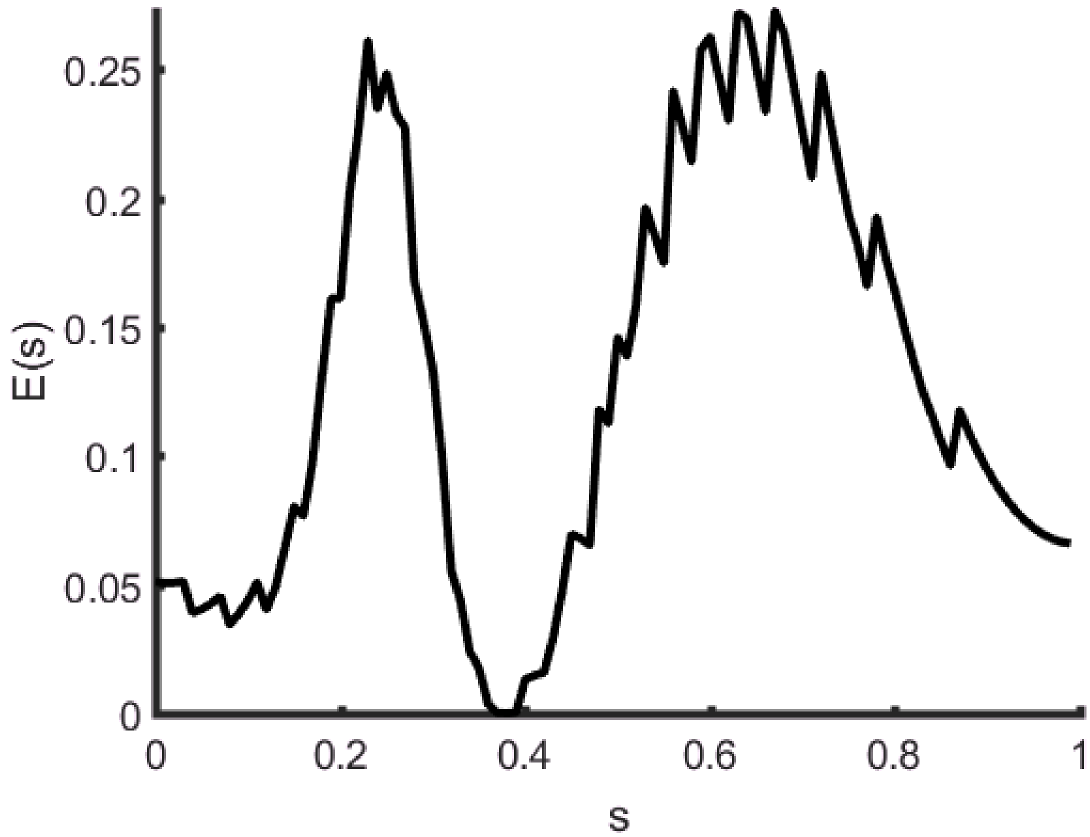
```
% gda14_01
% supports Figure 14.1
% graphical interpretation of Lagrange Multipliers

clear all;

% (x,y) grid
N=51;
M=51;
Dx=0.01;
Dy=0.01;
x=Dx*[0:N-1]';
y=Dy*[0:M-1]';

% Funtion E(x,y) that is minimized, a long-wavelngth sinusoid
E=-sin(x)*sin(y)';

%  gradient of E
dEdx = -cos(x)*sin(y)';
dEdy = -sin(x)*cos(y)';

% gradient list, for plotting purposed
gs = 0.1;  % scale factor
%      a   b   b   b   c   c  d   d   d, e  e  e  e
gi = [40, 30, 40, 25, 40, 10, 20, 10, 15, 4,  5, 8, 12]';
gj = [40, 30, 25, 40, 10, 40, 10, 24, 18, 16, 10, 6, 3]';
Ng = length(gi);

% plot E(x,y)
figure(1);
clf;
colormap('jet');
set(gca,'LineWidth',3);
hold on;
set(gca,'FontSize',14);
axis ij;
axis( [x(1), x(N), y(1), y(M)] );
imagesc( [x(1), x(N)], [y(1), y(M)], E );

% plot direction of gradient at selected points
sn = 0.04;
for k=[1:Ng]
    i = gi(k);
    j = gj(k);
    g = [dEdx(i,j), dEdy(i,j)]';
    g = g/sqrt(g'*g);
    plot( x(i), y(j), 'wo', 'LineWidth', 3 );
    plot( [x(i),x(i)+sn*g(1)]', [y(j),y(j)+sn*g(2)]', 'w-', 'LineWidth', 3 );
end

% constraint c(x,y)=0 is a parametric curve [xc(s), yc(s)]
% where the parameter s varies from 0 to 1 along curve
% I specify the curve, not the constraint itself
Ns = 101;
s = [0:Ns-1]'/(Ns-1);
% endpoints
cx1 = 0.00;
cy1 = 0.38;
cx2 = 0.25;
```

```
cy2 = 0.00;
% line connecting endpoints
xc = (1-s)*cx1 + s*cx2;
yc = (1-s)*cy1 + s*cy2;
% embelish line into a curve
yc = yc - 0.1*sin(pi*s);
yc = yc - 0.05*exp( -((s-0.1).^2)/(2*0.1*0.1) );
yc = yc - 0.03*exp( -((s-0.2).^2)/(2*0.1*0.1) );
plot( xc, yc', 'k-', 'LineWidth', 3);

% plot arrows on one side of curve
dxc = diff(xc); % dx/ds
dyc = diff(yc); % dy/ds
for k=[1:7:Ns-1]
    p = [xc(k), yc(k)]';   % point on curve
    t = [dxc(k), dyc(k)]'; % tangent to curve
    t = t / sqrt(t'*t);
    n = [t(2), -t(1)];     % normal to curve
    sn = 0.03;
    plot( [p(1),p(1)+sn*n(1)]', [p(2),p(2)+sn*n(2)]', '-', 'LineWidth',2, 'Color', [0.7,0.7,0.
end

% tabulate the deviation between n and gradE/|gradE| along
% the constraint curve
e = zeros(Ns-1,1); % deviation
ie = zeros(Ns-1,1);
je = zeros(Ns-1,1);
for k=[1:Ns-1] % tabulate deviation
    p = [xc(k), yc(k)]';   % point on curve
    i = floor(p(1)/Dx)+1;  % pixel of this point
    j = floor(p(2)/Dy)+1;
    t = [dxc(k), dyc(k)]'; % tangent to curve
    t = t / sqrt(t'*t);
    n = [t(2), -t(1)];     % normal to curve
    g = [dEdx(i,j), dEdy(i,j)]'; % gradient at this point
    g = g/sqrt(g'*g);      % direction of gradient
    e(k) = (n(1)-g(1))^2 + (n(2)-g(2))^2; % deviation
    ie(k) = i; % back-pointer i(k)
    je(k) = j; % back-pointer j(k)
end

% find and plot point where gradE and n are anti-parallel
[emin, k] = min(e);
i = ie(k);
j = je(k);
plot( x(i), y(j), 'ko', 'LineWidth', 4 );
```




```
% Figure 14.1 Graphical interpretation of the method of Lagrange multipliers,
% in which the function E(x, y) is minimized subject to the constraint that
% C(x, y) = 0. The solution (bold dot) occurs at the point (x0, y0) on the
% curve C(x, y) = 0, where the perpendicular direction (gray arrows) is parallel
% to the gradient ∇E(x, y) (white arrows). At this point, E can only be further
% minimized by moving the point (x0, y0) off of the curve, which is disallowed by
% the constraint.


% plot deviation along curve
figure(2);
clf;
set(gca,'LineWidth',3);
set(gca,'Fontsize',14);
hold on;
axis( [0, 1, 0, max(e) ] );
plot( s(1:Ns-1), e, 'k-', 'LineWidth', 3 );
xlabel('s');
ylabel('E(s)');
```




```
% Figure.  Function E aling the curve, as a function of arc-length s.
```