```
% gda04_05
%
% trade off of resolution and variance
% supports Figure 4.7

% auxially variable z
M=101;
zmin=0;
zmax=10;
Dz=(zmax-zmin)/(M-1);
z=zmin+Dz*[0:M-1]';

% experiment: exponential smoothing of model
N=80;
cmin=0.00;
cmax=0.10;
Dc=(cmax-cmin);
c = cmin + Dc*[0:N-1]';
G = exp(-c*z'); % data kernel

% Part 1: Backus-Gilbert
% I hand-tuned the points on trade-off curve to make the plot look nice
alphavec = [0.999, 0.995, 0.99, 0.5, 0.4, 0.3, 0.2, 0.1, 0.05, 0.01, 0.001, 0.0001, 0.00001]';
iahalf=find(alphavec==0.5);
Na=length(alphavec);   % number of points on the trade-off curve
spreadR1=zeros(Na,1);  % tabulate spread of resolution
sizeC1=zeros(Na,1);    % tabulate size of covariance

for a=[1:Na]

    alpha = alphavec(a);

    % construct Backus-Gilbert solution row-wise
    GMG = zeros(M,N);
    u = G*ones(M,1);
    for k = [1:M]
        S = G * diag(([1:M]-k).^2) * G';
        Sp = alpha*S + (1-alpha)*eye(N,N);
        uSpinv = u'/Sp;
        GMG(k,:) = uSpinv / (uSpinv*u); % generalized inverse
    end

    Cm1 = GMG*GMG'; % unit covariance matrix
    R1 = GMG*G;     % model resolution matrix
    sizeC1(a) = sum(diag(Cm1)); % size of covariacne

    % spread of resolution

    % the hard way, to check result
    % spreadR(a)=0;
    % for i = [1:M]
    % for j = [1:M]
    %    spreadR(a) = spreadR(a) + ((i-j)^2)*(R(i,j)^2);
    % end
    % end

    spreadR1(a) = sum(sum( (([1:M]'*ones(1,M)-ones(M,1)*[1:M]).^2).*(R1.^2) ));

end
```

```
% plot
figure(1);
clf;
subplot(1,2,1);

% plot trade-off curve
set(gca,'LineWidth',3);
set(gca,'FontSize',14);
hold on;
axis( [2, 3.5, -3, 5 ]' );
plot( log10(spreadR1), log10(sizeC1), 'k-', 'LineWidth', 4);
% plot( log10(spreadR1), log10(sizeC1), 'ro', 'LineWidth', 2);
plot( log10([spreadR1(1), spreadR1(iahalf), spreadR1(Na)]') , log10([sizeC1(1), sizeC1(iahalf)
xlabel('log10 spread(R)');
ylabel('log10 size(Cm)');

% Part 2: Damped minimum length
% I hand-tuned the points on trade-off curve to make the plot look nice
alphavec = [0.9999, 0.999, 0.995, 0.99, 0.5, 0.4, 0.3, 0.2, 0.1, 0.05, 0.01]';
iahalf=find(alphavec==0.5);
Na=length(alphavec);   % number of points on the trade-of curve
spreadR2=zeros(Na,1);  % tabulate spread of reolution
sizeC2=zeros(Na,1);    % tabulate size of covariance

for a=[1:Na]

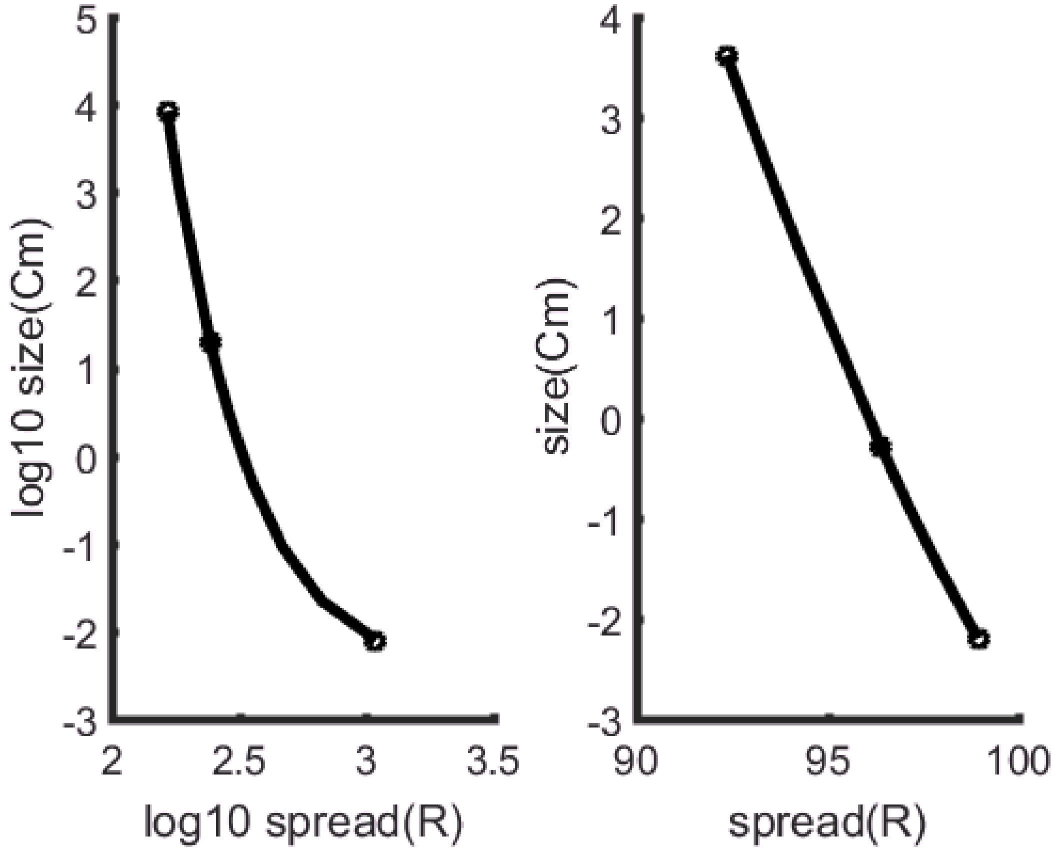
    alpha = alphavec(a);

    GMG = (alpha*G')/(alpha*(G*G')+ (1-alpha)*eye(N,N)); % generalized inverse

    Cm2 = GMG*GMG'; % model covariance
    R2 = GMG*G;     % model resolution
    sizeC2(a) = sum(diag(Cm2)); % size of covariance
    spreadR2(a) = sum(sum((R2-eye(M)).^2)); % spread of resulution

end

% plot trade-off curve
subplot(1,2,2);
set(gca,'LineWidth',3);
set(gca,'FontSize',14);
hold on;
axis( [90, 100, -3, 4 ]' );
plot( (spreadR2), log10(sizeC2), 'k-', 'LineWidth', 4);
% plot( (spreadR2), log10(sizeC2), 'ro', 'LineWidth', 2);
plot( ([spreadR2(1), spreadR2(iahalf), spreadR2(Na)]') , log10([sizeC2(1), sizeC2(iahalf), siz
xlabel('spread(R)');
ylabel('size(Cm)');
```

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% Figure 4.7 Trade-off curves of resolution and variance for the inverse problem shown in Figu
% (A) Backus-Gilbert solution, (B) damped minimum length solution. The larger the parameter α,
% more weight resolution is given (relative to variance) when forming the generalized inverse.
% details of the trade-off curve depend upon the parameterization. The resolution can be no be
% than the smallest element in the parameterization and no worse than the sum of all the eleme
.
% output tabulation to the command window
% [alphavec, spreadR2, log10(sizeC2)]
```