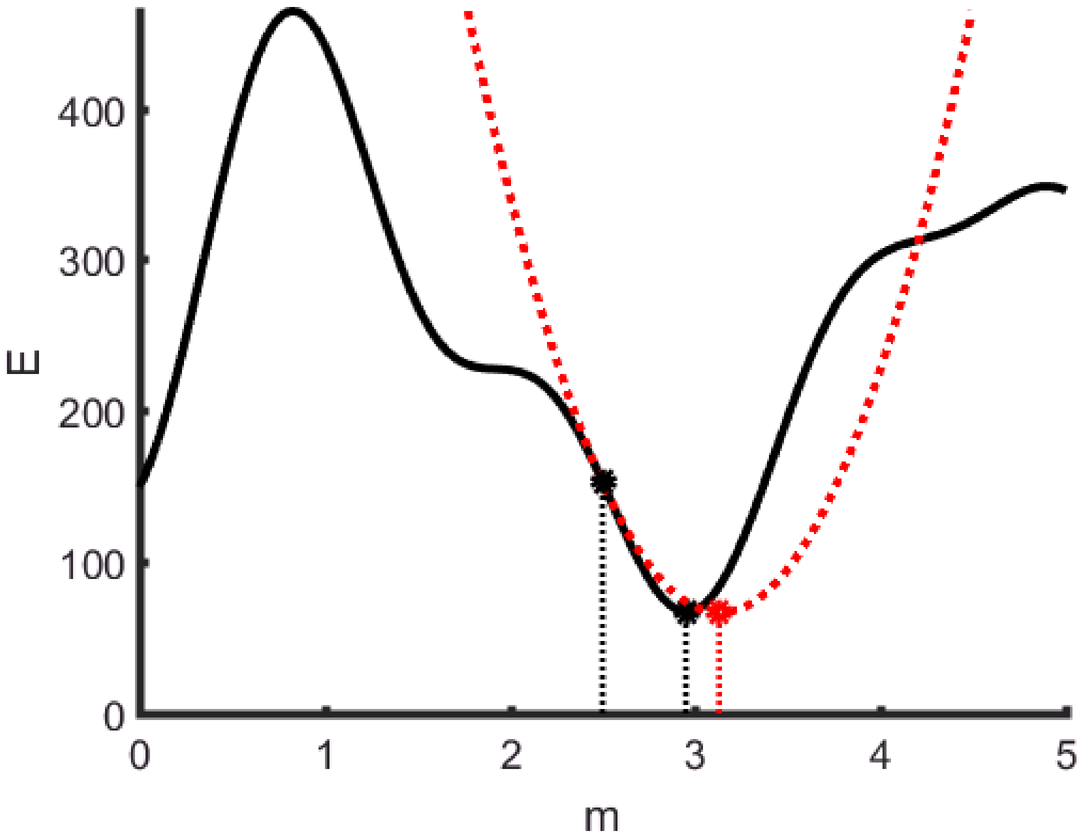
```
% gda09_09
%
% Newton's Method solution to exemplary 1D non-linear
% inverse problem, d(z) = sin(k*m*z)*z;
% This version uses a starting guess for the model
% parameter m1 that leads to convergence to the global
% minimum of the error surface (and hence to the correct
% value of the model parameter).
% Supports Figure 9.7

clear all;

% auxiliary parameter z
N = 21;
zmin = 0;
zmax = 5.0;
Dz = (zmax-zmin)/(N-1);
z = zmin + Dz*[0:N-1]';
zp = z.^0.5;

% define 1D grid for model parameter m1
Mg = 501;
mmin = 0;
mmax = 5;
Dm = (mmax-mmin)/(Mg-1);
m = mmin + Dm*[0:Mg-1];

% only one model parameter, m1
M=1;

% true model parameter
mtrue=3;

% true data
k=1;
dtrue = sin(k*mtrue.*zp).*z;

% observed data is true data plus random noise
sd=2;
dobs=dtrue+random('Normal',0,sd,N,1);

% tabulate error E(m) on the grid
E1=zeros(Mg,1);
for i=[1:Mg]
    dpre = sin(k*m(i).*zp).*z;
    e = dobs - dpre;
    E1(i) = (e'*e);
end

% find global minimum
[E1min, iE1min] = min(E1);
m1est=m(iE1min);

% Newton's method
% derivative
% d = sin(k*m(i).*zp).*z;
% dd/dm = k.*zp.*z.*cos(k*m(i).*zp);
% d = d(m0) + (dd/dm)|m0 (m-m0)
```

```
% tabulate a quadratic version of the error
% correspinding to a linearized version of
% the inverse problem (linearized around a point m0)
E2=zeros(Mg,1);
m0 = 2.5;
im0 = floor((m0-mmin)/Dm)+1;
d0 = sin(k*m0.*zp).*z;
dddm = k.*zp.*z.*cos(k*m0.*zp);
for i=[1:Mg]
    dpre = d0 + dddm * (m(i)-m0); % linearized approximation
    e = dobs - dpre;
    E2(i) = (e'*e);
end

% find minimum of error surface
[E2min, iE2min] = min(E2);
m2est=m(iE2min);

figure(1);
clf;
set(gca,'LineWidth',3);
set(gca,'FontSize',14);
hold on;
mminp=1; mmaxp=4;
axis( [mmin, mmax, 0, max(E1)] );
axis xy;

% true error surface, with circle at minimum, and line dropped to axis
plot( m, E1, 'k-', 'Linewidth', 3 );
plot( m1est, E1min, 'ko', 'Linewidth', 3 );
plot( [m1est, m1est], [0, E1min], 'k:', 'Linewidth', 2 );

% parabiolic approximation, with circle at minimum, and line dropped to
% axis
plot( m, E2, 'r:', 'Linewidth', 3 );
plot( m2est, E2min, 'ro', 'Linewidth', 3 );
plot( [m2est, m2est], [0, E2min], 'r:', 'Linewidth', 2 );

plot( m0, E2(im0), 'ko', 'Linewidth', 3 );
plot( [m0, m0], [0, E2(im0)], 'k:', 'Linewidth', 2 );

xlabel('m');
ylabel('E');
```




```
% Figure 9.7 The iterative method locates the global minimum mGM of the error E(m)
% (black curve) by determining the paraboloid (red curve) that is tangent to E at
% the trial solution  . The improved solution   is at the minimum (red circle) of
% this paraboloid and, under favorable conditions, can be closer to the solution
% corresponding to the global minimum than is the trial solution. MatLab script gda09_09.
```