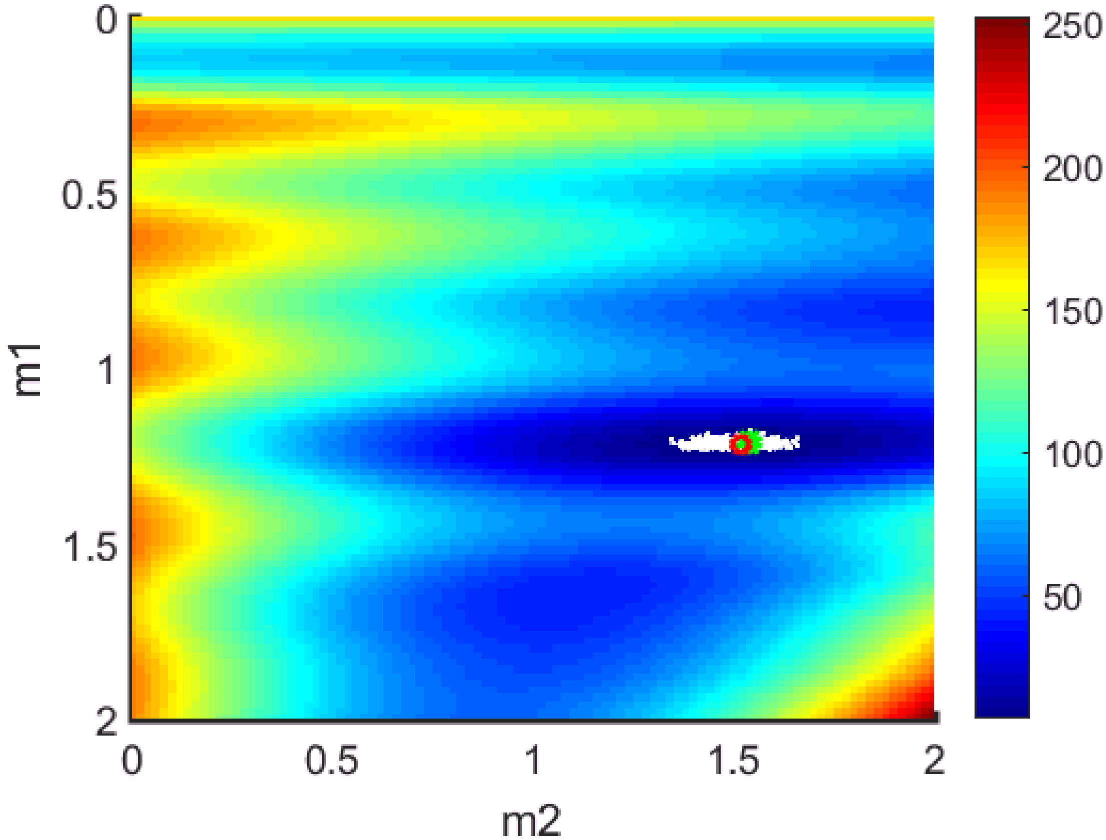
```
% gda09_20
% Bootstrap estimate of confidence interval
% for solution computed via Newton's method
% This function with these derivatives is being solved
% d(x) = sin( w0 m1  x) + m1 m2;
% dy/dm1 = w0 x cos( w0 m1 x) + m2
% dy/dm2 = m2
% supports Figure 9.18

% data are in an auxillary variable, x
N=40;
xmin=0;
xmax=1.0;
Dx=(xmax-xmin)/(N-1);
x = Dx*[0:N-1]';

% true model parameters
M=2;
mt = [1.21, 1.54]';

% y=f(x, m1, m2);
w0=20;
dtrue = sin(w0*mt(1)*x) + mt(1)*mt(2);
sd=0.4;
dobs = dtrue + random('Normal',0,sd,N,1);

% 2D grid, for plotting purposes only
L = 101;
Dm = 0.02;
m1min=0;
m2min=0;
m1a = m1min+Dm*[0:L-1]';
m2a = m2min+Dm*[0:L-1]';
m1max = m1a(L);
m2max = m2a(L);

% compute error, E, on grid for plotting purposed only
E = zeros(L,L);
for j = [1:L]
for k = [1:L]
    dpre = sin(w0*m1a(j)*x) + m1a(j)*m2a(k);
    E(j,k) = (dobs-dpre)'*(dobs-dpre);
end
end

% plot error surface
figure(1);
clf;
set(gca,'LineWidth',3);
hold on;
set(gca,'FontSize',14);
colormap('jet');
axis( [m2min, m2max, m1min, m1max] );
axis ij;
imagesc( [m2min, m2max], [m1min, m1max], E);
colorbar;
xlabel('m2');
ylabel('m1');
plot( mt(2), mt(1), 'go', 'LineWidth', 3 );
```

```
Nresamples = 1000;
m1save = zeros(Nresamples,1);
m2save = zeros(Nresamples,1);
for ir = [1:Nresamples]

 % resampling with duplications of data
 % (first estimate is without resampling)
    if( ir==1 )
        dresampled = dobs;
        xresampled = x;
    else
        rowindex = unidrnd(N,N,1);
        xresampled = x( rowindex );
        dresampled = dobs( rowindex );
    end

    % initial guess and corresponding error
    mg = [1.3, 1.5]';
    dg = sin(w0*mg(1)*xresampled) + mg(1)*mg(2);
    Eg = (dobs-dg)'*(dobs-dg);

    % iterate to improve initial guess
    Niter=100;
    for k = [1:Niter]

        dg = sin( w0*mg(1)*xresampled) + mg(1)*mg(2);
        dd = dresampled-dg;
        Eg=dd'*dd;
        Ehis(k+1)=Eg;

        G = zeros(N,2);
        G(:,1) = w0 * xresampled .* cos( w0 * mg(1) * xresampled ) + mg(2);
        G(:,2) = mg(2)*ones(N,1);

        % least squares solution
        dm = (G'*G)\(G'*dd);

        % update
        mg = mg+dm;
    end

    m1save(ir) = mg(1);
    m2save(ir) = mg(2);

    if( ir==1 )
        % plot estimated solution
        m1est = mg(1);
        m2est = mg(2);
        plot( m2est, m1est, 'ro', 'LineWidth', 2 );
    else
        % plot resampled solutions
        plot( mg(2), mg(1), 'w.', 'LineWidth', 2 );
    end

end

% plot again so is on top
plot( mt(2), mt(1), 'go', 'LineWidth', 3 );
plot( m2est, m1est, 'ro', 'LineWidth', 2 );
```

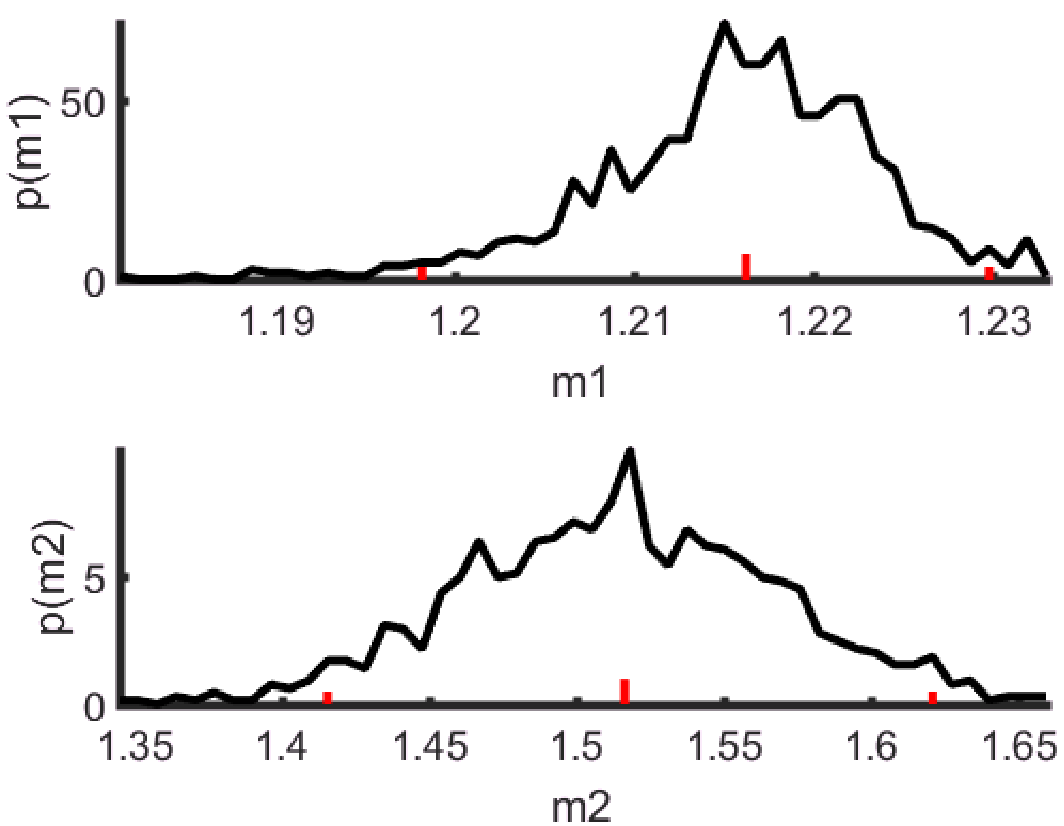



```
% Figure 9.18 Bootstrap confidence intervals for model parameters estimated
% using Newton's method for the same problem as in Figure 9.5. (A) Error surface
% (colors), showing true solution (red circle), estimated solution (green circle)
% and bootstrap solutions (white dots).

% histogram plot
figure(2);
clf;

% histogram2 for m1 and m2
Nbins=50;
m1hmin=min(m1save);
m1hmax=max(m1save);
Dm1bins = (m1hmax-m1hmin)/(Nbins-1);
m1bins=m1hmin+Dm1bins*[0:Nbins-1]';
m1hist = hist(m1save,m1bins);
pm1 = m1hist/(Dm1bins*sum(m1hist)); % norm al to pdf
Pm1 = Dm1bins*cumsum(pm1);
m1low=m1bins(find(Pm1>0.025,1));
m1high=m1bins(find(Pm1>0.975,1));
subplot(2,1,1);
set(gca,'LineWidth',3);
hold on;
set(gca,'FontSize',14);
xlabel('m1');
ylabel('p(m1)');
maxpm1=max(pm1);
axis( [m1hmin, m1hmax, 0, maxpm1] );
plot( m1bins, pm1, 'k-', 'LineWidth', 3 );
plot( [m1est, m1est]', [0 maxpm1/10]', 'r-', 'LineWidth', 3 );
plot( [m1low, m1low]', [0 maxpm1/20]', 'r-', 'LineWidth', 3 );
```

```
plot( [m1high, m1high]', [0 maxpm1/20]', 'r-', 'LineWidth', 3 );
fprintf('estimated m1: %f < %f < %f (95 percent confidence)\n', m1low, m1est, m1high );
```

```
estimated m1: 1.198161 < 1.216212 < 1.229757 (95 percent confidence)
```

```

m2hmin=min(m2save);
m2hmax=max(m2save);
Dm2bins = (m2hmax-m2hmin)/(Nbins-1);
m2bins=m2hmin+Dm2bins*[0:Nbins-1]';
m2hist = hist(m2save,m2bins);
pm2 = m2hist/(Dm2bins*sum(m2hist)); % norm al to pdf
Pm2 = Dm2bins*cumsum(pm2);
m2low=m2bins(find(Pm2>0.025,1));
m2high=m2bins(find(Pm2>0.975,1));

subplot(2,1,2);
set(gca,'LineWidth',3);
hold on;
set(gca,'FontSize',14);
xlabel('m2');
ylabel('p(m2)');
maxpm2=max(pm2);
axis( [m2hmin, m2hmax, 0, maxpm2] );
plot( m2bins, pm2, 'k-', 'LineWidth', 3 );
plot( [m2est, m2est]', [0 maxpm2/10]', 'r-', 'LineWidth', 3 );
plot( [m2low, m2low]', [0 maxpm2/20]', 'r-', 'LineWidth', 3 );
plot( [m2high, m2high]', [0 maxpm2/20]', 'r-', 'LineWidth', 3 );
```




```
% Figure 9.18 Bootstrap confidence intervals for model parameters estimated
% using Newton's method for the same problem as in Figure 9.5. (B) Empirically derived
```

```
% probability density function p(m1), with m1est (large red tick) and 95% confidence limits
% (small red ticks). (C) Same as (B), but for p(m2).

fprintf('estimated m2: %f < %f < %f (95 percent confidence)\n', m2low, m2est, m2high );
```

```
estimated m2: 1.415559 < 1.516381 < 1.620935 (95 percent confidence)
```