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% gda11_07
% supports Figure 11.10
% Uses heat flow problem to illustrate the use of
% the adjoint method to compute the data kernel dd/dm.
% This deriavtive is calculated two ways, using finite
% difference and using the adjoint method (achieving the
% same result, of course).

clear all;

figure(1);
clf;

% time t setup
N=101;
Dt = 0.1;
i0 = floor(N/10);
t = Dt*[-i0+1:N-i0]';

% reference solution is analytic
c0 = 0.7;
u0 = (t>0) .* exp( -c0*t );

% plot the reference solution
subplot(3,1,1);
set(gca,'LineWidth',3);
set(gca,'FontSize',14);
hold on;
axis( [t(1), t(end), -1.1, 1.1] );
xlabel('time t');
ylabel('u');

% the data is the b times the integral of the solution
% predicted data associated with the reference field
b = 0.5;
d0 = b*Dt*cumsum( (t>0) .* u0 );


% point scatterer at t0, that is
% c(t) = c0 + dm * delta(t-t0)
t0 = 0.5; % time of scatterer
it0 = find(t>=t0,1); % index of scatterer
dm = 0.3; % amplitude of scatterer
% field perturbation according the the Born approximation
du = -dm * u0(it0) * (t>t0) .* exp( -c0*(t-t0) );
u = u0 + du; % total field
d = b*Dt*cumsum( (t>0) .* u ); % predicted data

% plot the reference and total field
subplot(3,1,1);
set(gca,'LineWidth',3);
set(gca,'FontSize',14);
plot( t, u, 'k-', 'LineWidth', 3 );
plot( t, u0, 'r-', 'LineWidth', 2 );

% plot the predicted data associated with u and u0
% plot the data
subplot(3,1,2);
set(gca,'LineWidth',3);
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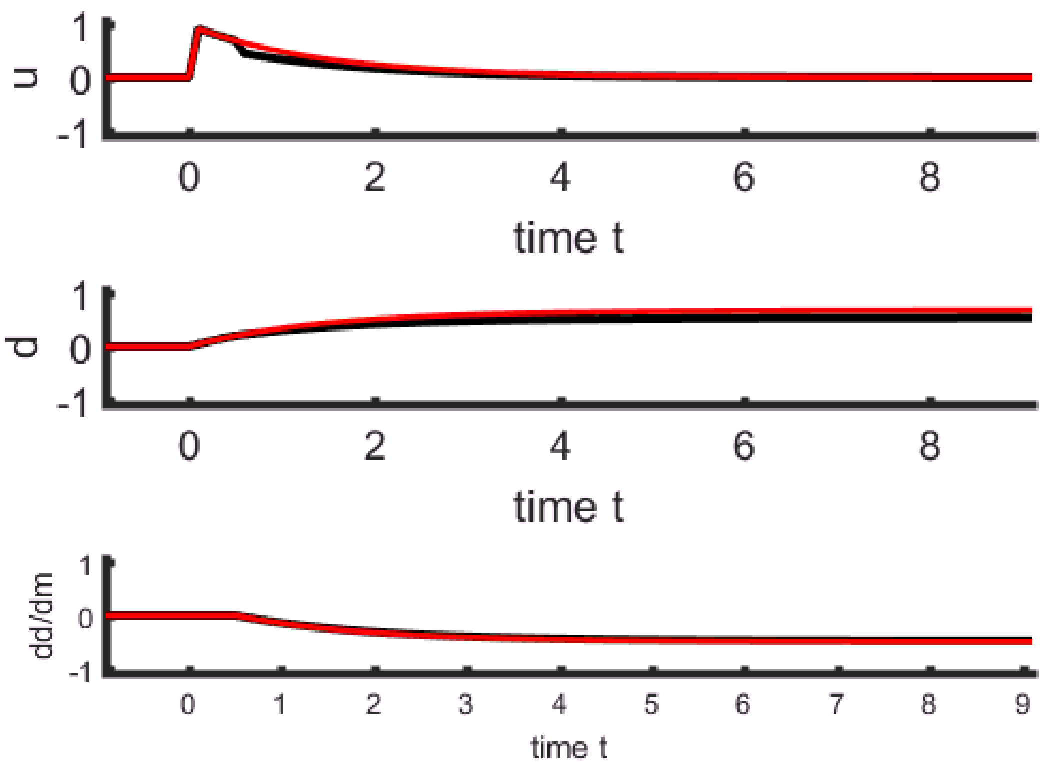
set(gca,'FontSize',14);
hold on;
axis( [t(1), t(end), -1.1, 1.1] );
xlabel('time t');
ylabel('d');
plot( t, d, 'k-', 'LineWidth', 3 );
plot( t, d0, 'r-', 'LineWidth', 2 );

% derivative by finite difference
dddm = (d-d0)/dm;
subplot(3,1,3);
set(gca,'LineWidth',3);
hold on;
yr = 1.1;
axis( [t(1), t(end), -yr, yr] );
plot( t, dddm, 'k-', 'LineWidth', 3 );
xlabel('time t');
ylabel('dd/dm');

% derivative by adjoint formula
% which should (and does) exactly match the adjoint formula
dddm2 = -(b/c0)*(t>0).*(t>t0).*(1-exp(-c0*(t-t0))).*exp(-c0*t0);
plot( t, dddm2, 'r-', 'LineWidth', 2 );
```

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% Figure 11.10. Data kernel for a perturbation in the material parameter in the heat flow prob
% (A) Reference field u_0 (t) (red curve) and perturbed field u(t)=u_0 (t)+δu(t) (red curve) for
% spike in the material parameter c_1 (t)=mδ(t-t_0 ) at time t_0 (dashed line). (B) Correspondin
% unperturbed (black) and perturbed data (red). (C) Data kernel calculated by finite differenc
% (black curve) and adjoint methods (red curve) match exactly. MatLab script gda11_07.
```