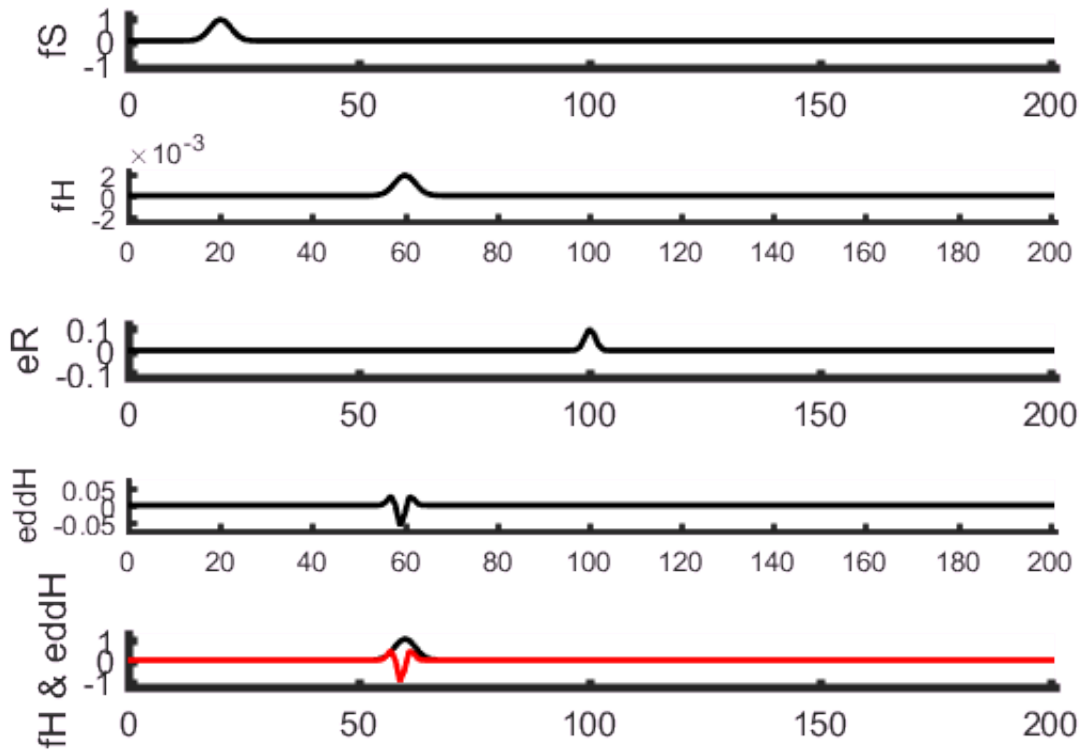
```
% gda13_02
% Support Figure 13.1
% Partial derivative of wavefield error for the
% acoustic case using adjoint methods.
% Supports Figure 13.1.

clear all;

% mycase controls time of error, 1, 2, 3 allowed
mycase = 2;

% background slowness
s0 = 1;
m = 1;

% independent variable x
Nx = 101;
Dx = 1;
x = Dx*[0:Nx-1]';
xmin = x(1);
xmax = x(end);

% independent variable y
Ny = 101;
Dy = 1;
y = Dy*[0:Ny-1]';
ymin = y(1);
ymax = y(end);

% independent variable t
Nt = 2*Nx;
Dt = 1;
t = Dt*[0:Nt-1]';
tmin=t(1);
tmax=t(end);

% source time
t0 = tmin + (tmax-tmin)/10;

% source time function is Gaussian curve
fS = exp( -0.1*(t-t0).^2 );

% source location
xS = 80;
yS = 51;

% receiver location
xR = 20;
yR = 51;

% heterogeneity location
xH = 51;
yH = 79;

% distance and traveltime from Source to Receiver
RSR = sqrt( (xS-xR)^2 + (yS-yR)^2 );
TSR = s0*RSR;
nSR = floor( TSR/Dt );
```

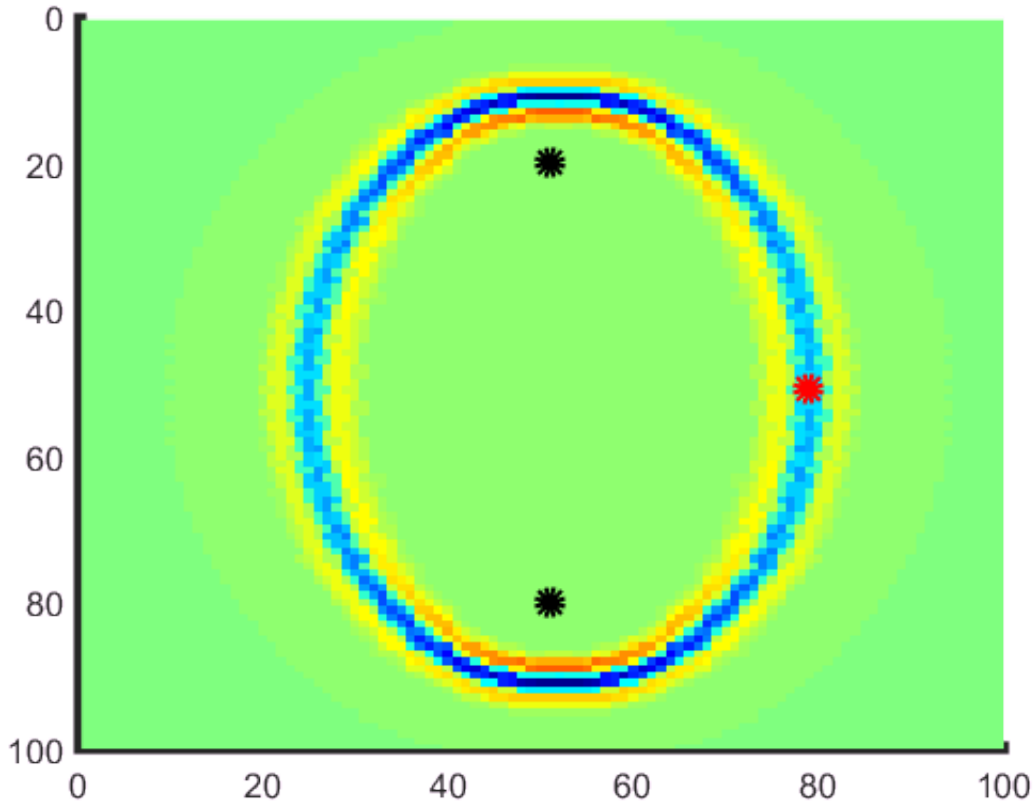
```
% distance and traveltime from Source to Heterogenety
RSH = sqrt( (xS-xH)^2 + (yS-yH)^2 );
TSH = s0*RSH;
nSH = floor( TSH/Dt );

% distance and traveltime from Heterogenety to Receiver
RHR = sqrt( (xH-xR)^2 + (yH-yR)^2 );
THR = s0*RHR;
nHR = floor( THR/Dt );

% reference field at the receiver
uR = zeros(Nt,1);
uR(nSR+1:Nt) = fS(1:Nt-nSR)/(4*pi*RSR);

% waveform error, a gaussian pulse at time te
e = zeros(Nt,1);
Ae = 0.1;
if( mycase == 1 )
    te = TSR+tmax/8;
elseif( mycase == 2 )
    te = TSR+tmax/5;
elseif( mycase == 3 )
    te = TSR+tmax/3.5;
end
e = e + Ae*exp( -0.4*(t-te).^2 );

% second derivative of error
edd = zeros(Nt,1);
edd(2:Nt-1) = (e(1:Nt-2)-2*e(2:Nt-1)+e(3:Nt))/(Dt^2);

NTPLOTS = 5;
figure(1)
clf;

% plot source time function
subplot(NTPLOTS,1,1);
set(gca,'LineWidth',3);
set(gca,'FontSize',12);
hold on;
axis( [tmin, tmax, -1.25*max(abs(fS)), 1.25*max(abs(fS))] );
plot( t, fS, 'k-', 'LineWidth', 2 );
ylabel('fS');

% plot reference wavefield at heterogeneiy
fH = zeros(Nt,1);
fH(nSH+1:Nt) = fS(1:Nt-nSH)/(4*pi*RSH);
subplot(NTPLOTS,1,2);
set(gca,'LineWidth',2);
hold on;
axis( [tmin, tmax, -1.25*max(abs(fH)), 1.25*max(abs(fH))] );
plot( t, fH, 'k-', 'LineWidth', 2 );
ylabel('fH');

% plot error
subplot(NTPLOTS,1,3);
set(gca,'LineWidth',3);
set(gca,'FontSize',12);
hold on;
axis( [tmin, tmax, -1.25*max(abs(e)), 1.25*max(abs(e))] );
plot( t, e, 'k-', 'LineWidth', 2 );
ylabel('eR');
```

```
% plot second derivarive of error at heterogeneity
eddH = zeros(Nt,1);
eddH(1:Nt-nHR) = edd(nHR+1:Nt);
subplot(NTPLOTS,1,4);
set(gca,'LineWidth',2);
hold on;
axis( [tmin, tmax, -1.25*max(abs(eddH)), 1.25*max(abs(eddH))] );
plot( t, eddH, 'k-', 'LineWidth', 2 );
ylabel('eddH');

% plot reference wavefiels and error at heterogeneity
subplot(NTPLOTS,1,5);
set(gca,'LineWidth',3);
set(gca,'FontSize',12);
hold on;
a = fH/max(abs(fH));
b = eddH/max(abs(eddH));
axis( [tmin, tmax, -1.25, 1.25] );
plot( t, a, 'k-', 'LineWidth', 2 );
plot( t, b, 'r-', 'LineWidth', 2 );
ylabel('fH & eddH');
```




```
% Figure 13.1. Partial derivative ∂E/∂m of the wave field error E(x_R ) for an acoustic
% wave propagation problem.  The source x_S and receiver x_R  (back circles) are separated
% by a distance of R=60 km. The perturbation in acoustic velocity is δv=(m/2v_0 ) δ(x-x_h ),
% where v_0=1 km/s is a constant reference velocity and x_h is the heterogeneity's location.
% The wave field error e(t) is a Gaussian pulse at time R⁄v_0 +τ.

d = zeros(Nx,Ny);
```

```
% tabulate dEdm on 2D grid
for ix = [1:Nx]
for iy = [1:Ny]

    % heterogeneity location
    xh = x(ix);
    yh = y(iy);

    % source to heterogeneity parameters
    RSh = sqrt( (xS-xh)^2 + (yS-yh)^2 + Dx^2);
    TSh = s0*RSh;
    NSh = floor(TSh/Dt);

    % heterogeneity to receiver parameters
    RhR = sqrt( (xR-xh)^2 + (yR-yh)^2 + Dx^2);
    ThR = s0*RhR;
    NhR = floor(ThR/Dt);

    % reference wavefield
    a = zeros(Nt,1);
    a(NSh+1:Nt) = fS(1:Nt-NSh)/(4*pi*RSh);

    % adjoint wavefield
    b = zeros(Nt,1);
    b(1:Nt-NhR) = 4*s0*edd(NhR+1:Nt)/(4*pi*RhR);

    % correlate the two wavefields
    d(ix,iy) = sum( a .* b );

end
end

% plot 2D inage of wavefield with source, heterogeneity,
% receiver positions superimposed
figure(2);
clf;
set(gca,'LineWidth',3);
set(gca,'FontSize',12);
colormap('jet');
hold on
axis ij
axis( [xmin, xmax, ymin, ymax] );
dmax = max(abs(d(:)));
imagesc(d,[-dmax,dmax]);
plot( yS, xS, 'ko', 'LineWIdth', 4 );
plot( yR, xR, 'ko', 'LineWIdth', 4 );
plot( yH, xH, 'ro', 'LineWIdth', 4 );
```




```
% Figure 13.1 (A) Partial derivative (colors) for a suite of x_h's covering
% the (x,y) plane, for τ=25,40 and 57 s, respectively.
```