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% gda07_05
% supports Figure 7.6
% graphical interpretation of Kuhn-Tucker theorem

clear all;

% (x,y) grid
N=51;
M=51;
Dx=0.01;
Dy=0.01;
x=Dx*[0:N-1]';
y=Dy*[0:M-1]';

% a long-wavelength, 2D sinusoid
E=-sin(x)*sin(y)';

% minus gradient
dEdx = cos(x)*sin(y)';
dEdy = sin(x)*cos(y)';

% gradient list
gs = 0.1; % scale factor
gi = [40, 30, 40, 25, 40, 10, 10, 20, 10]';
gj = [40, 30, 25, 40, 10, 40, 10, 10, 20]';
Ng = length(gi);

% plot
figure(1);
clf;
set(gca, 'LineWidth', 3);
hold on;
set(gca, 'FontSize', 14);
colormap('jet');
axis ij;
axis( [x(1), x(N), y(1), y(M)] );
imagesc( [x(1), x(N)], [y(1), y(M)], E );

for k=1:Ng
    i = gi(k);
    j = gj(k);
    plot( x(i), y(j), 'wo', 'LineWidth', 3 );
    plot( [x(i), x(i)+gs*dEdx(i,j)], [y(j), y(j)+gs*dEdy(i,j)]', 'w-', 'LineWidth', 3 );
end

% plot inequality constraint line
k1=7;
k2 = N-7+1;
plot( [x(k1), x(k2)]', [y(end), y(1)]', 'k-', 'LineWidth', 3);

% plot arrows on feasible side of equality
p1 = [x(k1), y(end)]';
p2 = [x(k2), y(1)]';
t = p1-p2;
t = t/sqrt(t'*t);
n = [-t(2), t(1)]';
n = n/sqrt(n'*n);
sn = 0.03;
Nn = 10;

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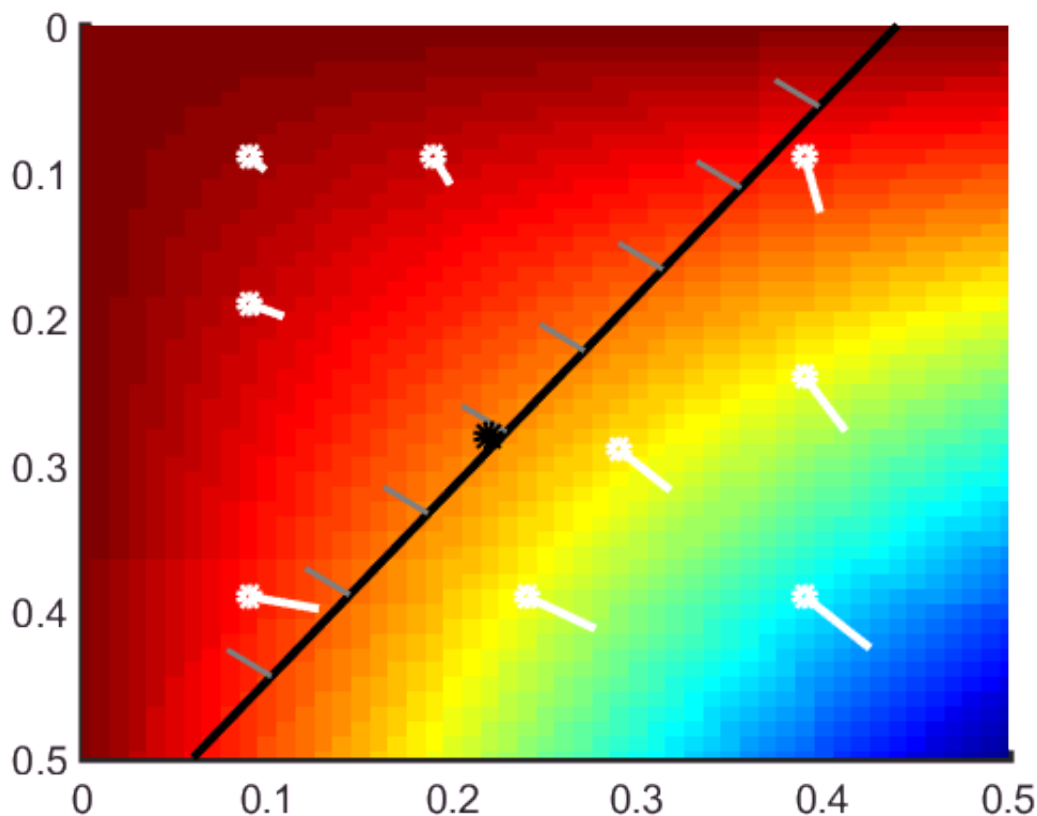
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for k=[2:Nn-1]
    a = (k-1)/(Nn-1);
    p = a*p1 + (1-a)*p2;
    plot( [p(1),p(1)+sn*n(1)]', [p(2),p(2)+sn*n(2)]', '-', 'LineWidth',2, 'Color', [0.5,0.5,0.5]);
end

% tabulate the deviation between n and -gradE/|gradE| along
% the constraint line
Nn = 101;
e = zeros(Nn,1);
ie = zeros(Nn,1);
je = zeros(Nn,1);
for k=[1:Nn]
    a = (k-1)/(Nn-1);
    p = a*p1 + (1-a)*p2;
    i = floor(p(1)/Dx)+1;
    j = floor(p(2)/Dy)+1;
    g = -[dEdx(i,j), dEdy(i,j)]';
    g = g/sqrt(g'*g);
    e(k) = (n(1)-g(1))^2 + (n(2)-g(2))^2;
    ie(k) = i;
    je(k) = j;
end

% find and plot point where gradE and n are anti-parallel
[emin, k] = min(e);
i = ie(k);
j = je(k);
plot( x(i), y(j), 'ko', 'LineWidth', 4 );

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% Figure 7.6 The error $E(m)$ (colors) has a single minimum (in lower right). The linear equality

% constraint, $H_m \geq h$, divides $S(m)$ into two half-spaces (black line, with gray arrows pointing
% into the feasible half-space). Solution (circle) lies on the boundary between the two
% half-spaces and therefore satisfies the constraint in the equality sense. At this point,
% the normal of the constraint hyperplane (gray arrow) is antiparallel to $-\nabla E$ (white arrows).