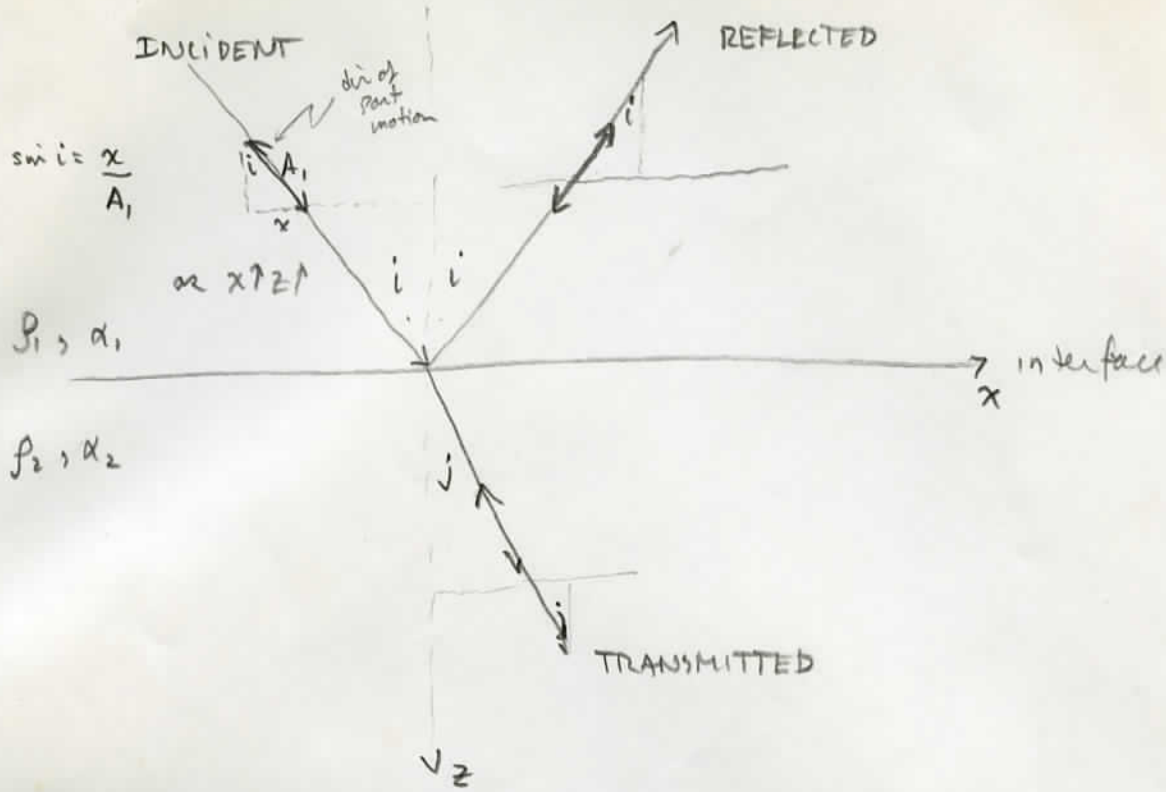
Plane wave incident on fluid - fluid interface



I choose to work in displacements. Then wave fcns for the 3 waves have the form:

$$u_I = A_1 \begin{pmatrix} \sin i \\ 0 \\ \cos i \end{pmatrix} \exp i\omega(px + \xi_1 z - t)$$

$$u_R = A_2 \begin{pmatrix} -\sin i \\ 0 \\ \cos i \end{pmatrix} \exp i\omega(px - \xi_1 z - t)$$

$$u_T = A_3 \begin{pmatrix} \sin j \\ 0 \\ \cos j \end{pmatrix} \exp i\omega(px + \xi_2 z - t)$$

note $\quad p = \frac{\sin i}{\alpha_1} = \frac{\sin j}{\alpha_2} \qquad \xi_1 = \frac{\cos i}{\alpha_1} = \frac{1}{\alpha_1}\sqrt{1 - \alpha_1^2 p^2}$

Boundary conditions : displacement ; Traction continuous at interface . This implies :

$$u_{x_I} + u_{x_R} = u_{x_T}$$

$$u_{zI} + u_{zR} = u_{zT}$$

$$\sigma_{33}(z=0^+) = \sigma_{33}(z=0^-)$$

and we have $\sigma_{ij} = c_{ijpq}\frac{1}{2}(u_{p,q}+u_{q,p})$ for a fluid $c = \alpha^2\rho\,\delta_{ij}\delta_{pq}$ so that conditions are

$$\sigma_{33} = \alpha^2\rho\,(u_{1,1}+u_{3,3})$$

these give :

$$\sin i \quad A_1 \quad - \quad \sin i \quad A_2 \quad = \quad \sin j \quad A_3$$

$$\cos i \quad A_1 \quad + \quad \cos i \quad A_2 \quad = \quad \cos j \quad A_3$$

$$\alpha_1^2\rho_1\left[p\sin i + \xi_1\cos i\right]A_1 + \alpha_1^2\rho_1\left[p\sin i - \xi_1\cos i\right]A_2 = \alpha_2^2\rho_2\left[p\sin j + \xi_2\cos j\right]A_3$$

for the simple case of normal incidence
$\phi = \theta = 0$, $p=0$ $\xi_1 = \frac{1}{\alpha_1}$ $\xi_2 = \frac{1}{\alpha_2}$ and eqns become

$$0 + 0 = 0$$

$$A_1 + A_2 = A_3 \qquad \therefore \quad A_2 = A_3 - A_1$$

$$\alpha_1\rho_1 A_1 - \alpha_1\rho_1 A_2 = \alpha_2\rho_2 A_3 \qquad \therefore \quad A_1 - A_2 = \frac{\alpha_2\rho_2}{\alpha_1\rho_1}A_3$$

$$2A_1 \cancel{-A_3} = \left[\frac{\alpha_2\rho_2}{\alpha_1\rho_1}+1\right]A_3 \qquad \frac{A_3}{A_1} = 2\left[\frac{\alpha_2\rho_2}{\alpha_1\rho_1}+\frac{\alpha_1\rho_1}{\alpha_1\rho_1}\right]$$

$$= 2\left(\frac{\alpha_1\rho_1+\alpha_2\rho_2}{\alpha_1\rho_1}\right)$$

$$A_1 - A_2 = \frac{\rho_2 \alpha_2}{\rho_1 \alpha_1} \left[A_1 + A_2 \right]$$

$$\left[1 - \frac{\rho_2 \alpha_2}{\rho_1 \alpha_1}\right] A_1 = + \left[1 + \frac{\rho_2 \alpha_2}{\rho_1 \alpha_1} \right] A_2$$

$$\frac{\rho_1 \alpha_1 - \rho_2 \alpha_2}{\cancel{\rho_1 \alpha_1}} A_1 = + \left[\frac{\rho_1 \alpha_1 + \rho_2 \alpha_2}{\cancel{\rho_1 \alpha_1}} \right] A_2$$

$$\frac{A_2}{A_1} = + \frac{\rho_1 \alpha_1 - \rho_2 \alpha_2}{\rho_1 \alpha_1 + \rho_2 \alpha_2}$$

(these check with my classnotes)