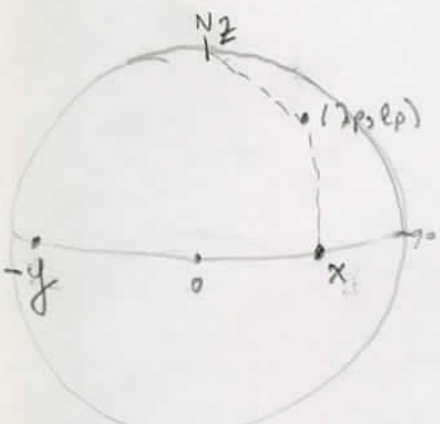
How to move a pole

1. given point (latitude, longitude) $= (\lambda, l)$, colatitude $= c = 90 - \lambda$ on sphere, what is the new (latitude, longitude) if the pole is moved to a new position (say (λ_p, l_p), c_p) in the following way.

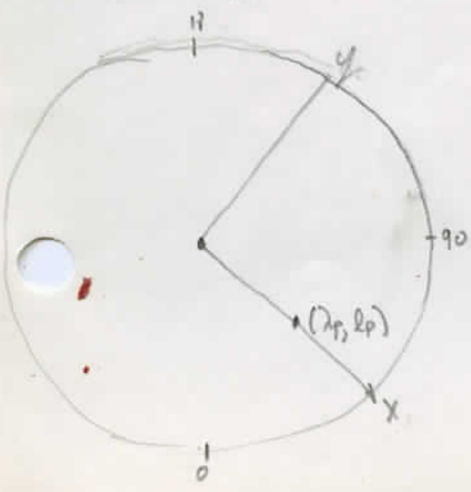
2. the pole is simply pushed over to the point (λ_p, l_p) and then given a counter-clockwise (when seen outside sphere above N. pole) twist of an angle T.

3. first switch to cartesian coordinate system which has z axis going through north pole and x axis going thru same meridian as new pole.



$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \sin c \; \cos(l - l_p) \\ \sin c \; \sin(l - l_p) \\ \cos c \end{pmatrix}$$

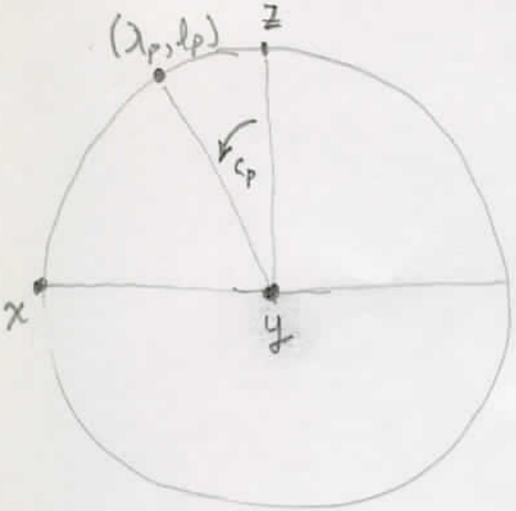
SIDE View



$$\begin{pmatrix} c \\ l \end{pmatrix} = \begin{pmatrix} \tan_2^{-1}\left(z, \sqrt{x^2 + y^2}\right) \\ \tan_2^{-1}(y, x) + l_p \end{pmatrix}$$

4. so take the point (λ, l) and switch it to (x, y, z)

then perform a rotation (counterclockwise) around y of C_p degrees followed by a rotation around the new z axis of T.



5. rotation around y

$$\begin{pmatrix} \cos C_p & 0 & -\sin C_p \\ 0 & 1 & 0 \\ \sin C_p & 0 & \cos C_p \end{pmatrix}$$

rotation around z

$$\begin{pmatrix} \cos T & \sin T & 0 \\ -\sin T & \cos T & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

product

$$\begin{pmatrix} \cos T \cos C_p & \sin T & -\cos T \sin C_p \\ -\sin T \cos C_p & \cos T & \sin T \sin C_p \\ \sin C_p & 0 & \cos C_p \end{pmatrix} = ROT$$

6. the pushing over followed by twisting is accomplished by multiplying

$$X_{new} = (ROT)\, X_{OLD}$$

$$X_{OLD} = (ROT)^T X_{new}$$

7. after rotating transform back to (λ, ℓ) by reverse procedure given in 3.

Error estimation

1. ~~First~~ st estimate the standard deviation of the problem from the goodness of fit of the data.

$$\varepsilon_i = \tan \lambda_i + \tan \varphi_p \cos (l_i - l_p) = \text{error}$$

$$\sigma^2 = \text{STD DEVIATION} = |\Sigma \varepsilon_i|^2 / (m-k)$$

m = # of observations

k = # of non zero eigenvalues

2. then compute the SVD decomposition of the matrix of partial derivatives. let the eigenvector matrix be V (V should be 2x2)

$$\sigma_i^2 = \sigma^2 \sum_{j=1}^{k} V_{ij}^2 / \lambda_j^2 = \text{STD Deviation of } i\text{th variable.}$$

$$\sigma_1 = \sqrt{\sigma^2 (V_{11}^2 / \lambda_1^2 + V_{12} / \lambda_2^2)}$$

$$\sigma_2 = \sqrt{\sigma^2 (V_{21}^2 / \lambda_1^2 + V_{22} / \lambda_2^2)}$$