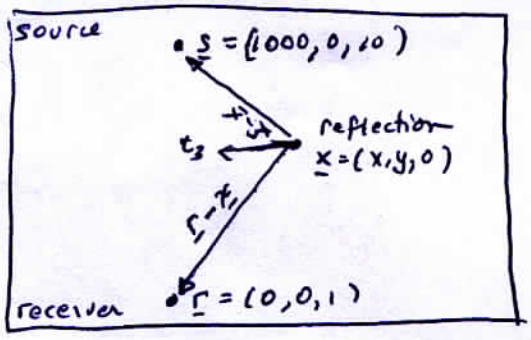
MRN

6-7-99 Paul Richards' river reflection problem

What makes "elliptical" reflections of lights on the rippled surface of a lake?

```
#include <stdio.h>
#include <math.h>
#define DTOR (3.141592654/180.0)
main(argc, argv)
int argc;
char *argv[];
{
        int i, j, k;
        double s[3], r[3], x[3];
        double t1[3], t2[3], t3[3], l, theta, test;

        sscanf(argv[1], "%le", &test );

        s[0]=1000.0; s[1]=0.0; s[2]=10.0;
        r[0]=0.0;    r[1]=0.0; r[2]= 1.0;

        printf("x\ty\n");

        for( i=1; i<1000; i++ ) { for( j=100; j>=0; j-- ) {
                x[0]=(double)i; x[1]=(double)j/10.0; x[2]=0.0;
                l=0.0; for( k=0; k<3; k++ ) { t1[k]=s[k]-x[k]; l+=t1[k]*t1[k]; }
                l = sqrt(l); for( k=0; k<3; k++ ) t1[k]/=l;
                l=0.0; for( k=0; k<3; k++ ) { t2[k]=r[k]-x[k]; l+=t2[k]*t2[k]; }
                l = sqrt(l); for( k=0; k<3; k++ ) t2[k]/=l;
                l=0.0; for( k=0; k<3; k++ ) { t3[k]=0.5*(t1[k]+t2[k]); l+=t3[k]*t3[k]; }
                l = sqrt(l); for( k=0; k<3; k++ ) t3[k]/=l;
                theta = acos(t3[2])/DTOR;
                if( (theta<=test) || (x[0]==0.0) ) { printf("%f\t%f\n", x[0], x[1] ); break;
                }
        }
        exit(0);
        }
```

map view



unnormalized

$$\underline{t}_3 = \frac{1}{2}\frac{\underline{s}-\underline{x}}{|\underline{s}-\underline{x}|} + \frac{1}{2}\frac{(\underline{r}-\underline{x})}{|\underline{r}-\underline{x}|} \qquad \hat{t}_3 \leftarrow \frac{\underline{t}_3}{|\underline{t}_3|}$$

angle between $\hat{t}_3$ and vertical

$$\theta = \cos^{-1}(\hat{t}_3 \cdot \hat{z})$$

For a light ray to reflect off a point $x = (x, y, 0)$ on the lake, the surface normal at that point on the lake must have the right orientation for Snell's law to be satisfied. The required surface normal $\hat{n}$ is the average of the incident and reflected ray directions. If t_3 is inclined from vertical more than the rippling allows, then that point cannot reflect light. So find the locus of points on the lake surface where the normal is inclined less than some specified amount. This region is lit up.

envelope of
locus of points
with $\hat{t}_3$ within n degrees of vertical