with Value 8-1-7   MRN066   symmetric 2x2   (1)

tensor invariants and eigenvalues, given tensor e_{ij} (2x2 case)

let $A = e_{ii}$ $\quad B = e_{ij} e_{ij}$ $\quad \lambda$'s are eigenvalues

$$A = e_{11} + e_{22} = \lambda_1 + \lambda_2$$

$$B = e_{11}^2 + e_{12}^2 + e_{21}^2 + e_{22}^2 = \lambda_1^2 + \lambda_2^2 = e_{11}^2 + e_{22}^2 + 2 e_{12}^2$$

<u>first</u> λ's as function of A, B's

$$\lambda_2 = A - \lambda_1 \qquad \& \quad \lambda_1 = A - \lambda_2$$

$$\lambda_1^2 + (A - \lambda_1)^2 = B$$

$$\lambda_1^2 + A^2 + \lambda_1^2 - 2A\lambda_1 = B$$

$$2\lambda_1^2 - 2A\lambda_1 + (A^2 - B) = 0$$

$$2(A - \lambda_2)^2 - 2A(A - \lambda_2) + (A^2 - B) = 0$$

$$2A^2 + 2\lambda_2^2 - 4A\lambda_2 - 2A^2 + 2A\lambda_2 + A^2 - B = 0$$

$$2\lambda_2^2 - 2A\lambda_2 + (A^2 - B) = 0$$

note same eqn as for λ_1
eg. quadratic w/ 2 roots

<u>second</u> λ's in terms of components of tensor

$$\det\begin{pmatrix} e_{11} - \lambda & e_{12} \\ e_{12} & e_{22} - \lambda \end{pmatrix} = (e_{11} - \lambda)(e_{22} - \lambda) - e_{12}^2 = 0$$

$$= \lambda^2 - (e_{11} + e_{22})\lambda + e_{11} e_{22} - e_{12}^2$$

but $e_{11} + e_{22} = A$ and
$$A^2 - B = e_{11}^2 + e_{22}^2 + 2 e_{11} e_{22} - e_{11}^2 - e_{22}^2 - 2 e_{12}^2$$
$$= 2(e_{11} e_{12} - e_{12}^2)$$

so $\det(\cdots) = 0 = 2\lambda^2 - 2A\lambda + (A^2 - B)$

This is the same equation as in <u>first</u>

Thus solving for eigenvalues as functions of tensor invariants yields nothing <u>new</u> over solving traditional eigenvalue equation $\det(\underset{=}{e} - \lambda \underset{=}{I}) = 0$

~~Add'l max shear~~ $\cancel{D = 1/2(\lambda_1 - \lambda_2)}$

$\cancel{B = \lambda_1^2 + \lambda_2^2 = (\lambda_1 + \lambda_2)(\lambda_1 - \lambda_2) = AC}$

so ~~C = B/A can be written as fcn of tensor identities~~

third

max shear stress proportional to $D = \lambda_1 - \lambda_2$ (2)

λ : $a\lambda^2 + b\lambda + c = 0$

$$\lambda = -\frac{b}{2a} \pm \frac{\sqrt{b^2-4ac}}{2a}$$

$a = 2$
$b = -2A$
$c = A^2 - B$

so $\lambda_1 - \lambda_2 = \frac{1}{a}\sqrt{b^2-4ac} = \sqrt{\frac{b^2}{a^2} - 4\frac{c}{a}}$

with $\frac{b^2}{a^2} = \frac{4A^2}{4} = A^2$

$\frac{4c}{a} = 2(A^2-B)$ $\qquad \frac{b^2}{a^2} - \frac{4c}{a} = A^2 - 2A^2 + 2B = 2B - A^2$

so $\lambda_1 - \lambda_2 = \sqrt{2B - A^2} = D$ so D is a function of invariants A, B

fourth This checks

$$2B - A^2 = 2\lambda_1^2 + 2\lambda_2^2 - (\lambda_1 + \lambda_2)^2$$
$$= 2\lambda_1^2 + 2\lambda_2^2 - \lambda_1^2 - \lambda_2^2 - 2\lambda_1\lambda_2$$
$$= \lambda_1^2 + \lambda_2^2 - 2\lambda_1\lambda_2 = (\lambda_1 - \lambda_2)^2$$

fifth

Note if $A = 0$ (e.g. a deviatoric stress tensor)
then $D = 2B$.

3x3 case

$$A = \lambda_1 + \lambda_2 + \lambda_3 \qquad \lambda_1 + \lambda_2 = A - \lambda_3$$

$$B = \lambda_1^2 + \lambda_2^2 + \lambda_3^2$$

$$C = \lambda_1 \lambda_2 \lambda_3 \qquad \lambda_1 \lambda_2 = \frac{C}{\lambda_3}$$

$$A^2 = \lambda_1^2 + \lambda_2^2 + \lambda_3^2 + 2\lambda_1\lambda_2 + 2\lambda_1\lambda_3 + 2\lambda_2\lambda_3$$

$$A^2 = B + \frac{2C}{\lambda_3} + 2\lambda_3(\lambda_1 + \lambda_2)$$

$$A^2 = B + \frac{2C}{\lambda_3} + 2\lambda_3(A - \lambda_3)$$

$$0 = B + \frac{2C}{\lambda_3} + 2\lambda_3(A - \lambda_3) - A^2$$

$$0 = -2\lambda_3^3 + 2A\lambda_3^2 + (B - A^2)\lambda_3 + 2C$$

$$0 = \lambda_3^3 \underbrace{- A}_{p}\lambda_3^2 + \underbrace{\tfrac{1}{2}(A^2 - B)}_{q}\lambda_3 \underbrace{- C}_{r}$$

$$3a = 3q - p^2 = \frac{3}{2}(A^2 - B) - A^2 = \frac{A^2}{2} - \frac{3B}{2}$$

$$27b = 2p^3 - 9pq + 27r = -2A^3 + \frac{9A}{2}(A^2 - B) \mp 27C$$

3×3 case with A=0

$$\lambda^3 \underbrace{- \tfrac{1}{2}B\lambda}_{a} \underbrace{- C}_{b} = 0$$

a key quantity is $\frac{b^2}{4} + \frac{a^3}{27} = \frac{C^2}{4} - \frac{B^3}{27\cdot 8}$

$A = 0 = \lambda_1 + \lambda_2 + \lambda_3$; $-\lambda_3 = (\lambda_1 + \lambda_2)$

$B = 2\lambda_1^2 + 2\lambda_2^2 + 2\lambda_1\lambda_2$

$C = -\lambda_1\lambda_2(\lambda_1 + \lambda_2)$

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try to solve $(\lambda_1 - \lambda_2)$ and $(\lambda_1 + \lambda_2)$ as fcns of B, C.

get

$$\frac{B}{2} = (\lambda_1 - \lambda_2)^2 - \frac{3C}{\lambda_1 + \lambda_2}$$

$$\frac{B}{2} = (\lambda_1 + \lambda_2)^2 + \frac{C}{\lambda_1 + \lambda_2}$$

so if one could solve 2nd eqn for $(\lambda_1 + \lambda_2)$ then sub it into first. But 2nd is a cubic in $(\lambda_1 + \lambda_2)$.