schroendiger eqn $\dot{a} = -i\, a_{xx}$ $\left(\dot{a}^* = i\, a^*_{xx}\right)$ with BC $a \to 0$, $a_x \to 0$ at $\pm\infty$
(const. absorbed into time) $\dot{a} = da/dt$ $a_x = da/dx$

$$\frac{dP}{dt} = \frac{d}{dt}\int_{-\infty}^{+\infty} a^* a\, dx = \int_{-\infty}^{+\infty} (\dot{a}^* a + \dot{a} a^*)\, dx$$

$$= \int_{-\infty}^{+\infty} (i a^*_{xx} a - i a_{xx} a^*)\, dx$$

$$= i\int_{-\infty}^{+\infty} (a^*_{xx} a - a_{xx} a^*)\, dx$$

$$= i\int_{-\infty}^{+\infty} (a^*_{xx} a - [a^*_{xx} a]^*)\, dx$$

$$= i\int_{-\infty}^{+\infty} 2i\, \mathrm{Im}\,(a^*_{xx} a)\, dx$$

$$= -2\int_{-\infty}^{+\infty} \mathrm{Im}\,(a^*_{xx} a)\, dx = -2\,\mathrm{Im}\int_{-\infty}^{+\infty} a^*_{xx} a\, dx$$

upshot:
$p = a^* a$
conserves
total prob.
$P = \int p\, dx$
while
$p = (a^* a)^2$
does not

note:
$a_r + a_i i$
$-(a_r - a_i i)$
$= 2 a_i i$

$$\frac{d}{dx}(a^*_x a) = a^*_{xx} a + a^*_x a_x \quad \text{by chain rule}$$

$$a^*_{xx} a = \frac{d}{dx}(a^*_x a) + \underbrace{a^*_x a_x}_{\text{real, so has no imag part}}$$

$$\frac{dP}{dt} = -2\,\mathrm{Im}\int_{-\infty}^{+\infty} \frac{d}{dx}(a_x^* a)\, dx = -2\,\mathrm{Im}\left[\, a_x^* a \,\Big|_{-\infty}^{+\infty} \right] = 0$$

By B.C.'s

but if $P = \int_{-\infty}^{+\infty} (a^* a)^2 dx$

then $\frac{dP}{dt} = \frac{d}{dt}\int_{-\infty}^{+\infty} (a^* a)^2 dx = 2\int_{-\infty}^{+\infty} (a^* a)[\dot{a}^* a + a^* \dot{a}]\, dx$

$$= 2i\int_{-\infty}^{+\infty} (a^* a)[a^*_{xx} a - a^* a_{xx}]\, dx = 4i\,\mathrm{Im}\int_{-\infty}^{+\infty} (a^* a) a^*_{xx} a\, dx$$

$$= 4i\,\mathrm{Im}\int_{-\infty}^{\infty} (a^* a)\frac{d}{dx}(a^*_x a)\, dx - 4i\,\mathrm{Im}\int_{-\infty}^{\infty} \underbrace{(a^* a)(a^*_x a_x)}_{\text{real so has no imag part}}\, dx$$

$$= 4i\,\mathrm{Im}\int_{-\infty}^{\infty} (a^* a)\frac{d}{dx}(a^*_x a)\, dx \neq 0$$

note: integrand not equal to its conjugate $(a^* a)\frac{d}{dx}(a_x a^*)$ so integrand not real. Integrand not a total derivative so it is not just a fcn of its b.c.'s.