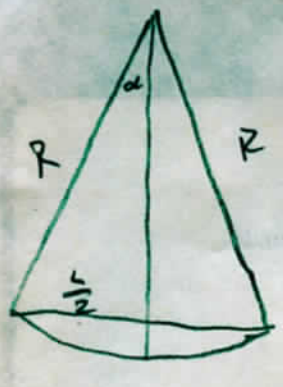
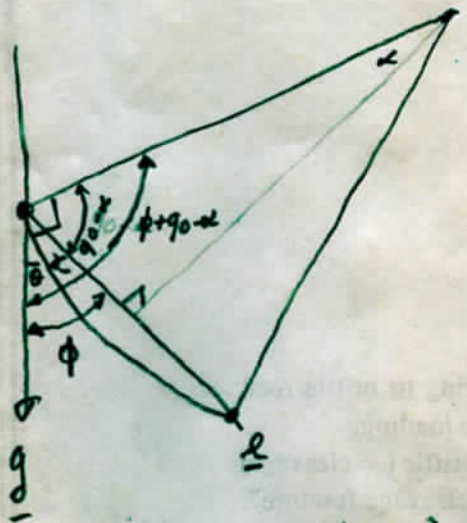
Perturbation of straight line ray path

5/7/01



$\sin\alpha = \frac{L}{2R}$

$\sin(a-b) = \sin a \cos b - \cos a \sin b$



$\cos\phi = \frac{\underline{g}\cdot\underline{\ell}}{|\underline{\ell}||\underline{g}|}$

$\theta + 90 = \phi + 90 - \alpha$

$\theta = \phi - \alpha$

$R = \frac{1}{P|g|} = \frac{V}{|g|\sin\theta}$

$\gamma = \phi + 90 - \alpha - (90-\alpha) - \theta$
$= \phi + 90 - \alpha - 90 + \alpha - \theta$
$= \phi - \theta$
$= \alpha$

$\sin\alpha = \frac{L}{2R}$

$\sin\alpha = \frac{L}{2}\frac{|g|\sin\theta}{V} = \frac{L|g|}{2V}\sin(\phi-\alpha)$ ← exact

for 1st order in small α

$k = \frac{L|g|}{2V}$

$\sin\phi - \alpha$
$= \sin\phi\cos\alpha - \cos\phi\sin\alpha$
$\approx \sin\phi - \alpha\cos\phi$

$\alpha \approx k(\sin\phi - \alpha\cos\phi)$

$\alpha = k\sin\phi - (k\cos\phi)\alpha$

$\alpha + k\cos\phi\,\alpha = k\sin\phi$

$\alpha(1 + k\cos\phi) = k\sin\phi$

$\alpha = \frac{k\sin\phi}{1 + k\cos\phi}$