Definition of Power Spectral Density in Two Dimensions
(getting the normalization right)

Bill Menke, December 2015

```
% 2D Power Sprectral Density
%
% W Menke, December 2015
%
% d = random('Normal', 0, sigmad, Nx, Ny );
%       uncorrelated random data
% ftt = fft2(d)*DX*DY;   % fourier transform of data
% ftt = ftt(:,1:Nyo2p1); % keep only right half-plane
% s2 = (2/(X*Y))*(abs(ftt).^2); % power spectral density (psd)
% P2 = DKx*DKy*sum(sum(s2)); % integral of psd
%      Note (Kx, Ky) has units of per meter, not per radian
% P = sum(sum(d.*d))/N; % power (= variance) of data
% P = P2

clear all;

Nx = 1024; % number of data in x
Ny = 512; % number of data in y
N = Nx*Ny; % total number of data
Nxo2p1 = Nx/2 + 1;  % Nx/2+1
Nyo2p1 = Ny/2 + 1;  % Ny/2+1

DX = 1.7; % x increment
DY = 2.3; % y increment
X = Nx*DX; % x interval
Y = Ny*DY; % y interval
% in analogy to temporal case, we work with analog
% of frequency f, as contrasted to angular frequncy w
Kx = 1/(2*DX); % nyquist non-angular x wavenumber
Ky = 1/(2*DY); % nyquist non-angular y wavenumber
DKx = Kx / (Nx/2); % spacing in x wavenumber
DKy = Ky / (Ny/2); % spacing in y wavenumber

sigmad=3;  % standard deviation of data
sigmad2=sigmad^2; % variance of data
d = random('Normal', 0, sigmad, Nx, Ny ); % uncorrelated random data

% remove mean
dmean = sum(sum(d))/N;
d = d - dmean;

P = sum(sum(d.*d))/N; % power (= variance) of data

ftt = fft2(d)*DX*DY;   % fourier transform of data
ftt = ftt(:,1:Nyo2p1); % keep only right half-plane
s2 = (2/(X*Y))*(abs(ftt).^2); % power spectral density (psd)

P2 = DKx*DKy*sum(sum(s2)); % non-angular wavenumber integral of psd

% At this poing, P = P2 (or nearly so)

fprintf('sigmad2 %f  P  %f   P2   %f\n', sigmad2, P, P2 );
```