Area Under Envelope is Twice Power in Signal

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Hilbert transform:

$$(\mathcal{H}u)_t = \frac{1}{\pi t} * u(t)$$

$$\mathcal{H}^\dagger = \mathcal{H}^{-1} \quad \text{and} \quad \mathcal{H}^{-1} = -\mathcal{H} \quad \text{and} \quad \mathcal{H}^\dagger = -\mathcal{H}$$

Inner product:

$$\langle u, v \rangle = \int_{-\infty}^{+\infty} u(t)\, v(t)\, dt$$

Analytic signal

$$\mathcal{A}u = u + i\, \mathcal{H}u = (\mathcal{I} + i\mathcal{H})u$$

It's complex conjugate

$$(\mathcal{A}u)^* = u - i\, \mathcal{H}u = (\mathcal{I} - i\mathcal{H})u$$

Envelope

$$(\mathcal{A}u)^* \mathcal{A}u$$

Area under envelope

$$\int (\mathcal{A}u)^* \mathcal{A}u\, dt =$$

$$\langle (\mathcal{A}u)^*, \mathcal{A}u \rangle = \langle u - i\, \mathcal{H}u, u + i\, \mathcal{H}u \rangle =$$

$$\langle u, u \rangle + \langle \mathcal{H}u, \mathcal{H}u \rangle = \langle u, u \rangle + \langle \mathcal{H}^\dagger \mathcal{H}u, u \rangle = \langle u, u \rangle + \langle \mathcal{H}^{-1} \mathcal{H}u, u \rangle = \langle u, u \rangle + \langle u, u \rangle = 2\langle u, u \rangle$$