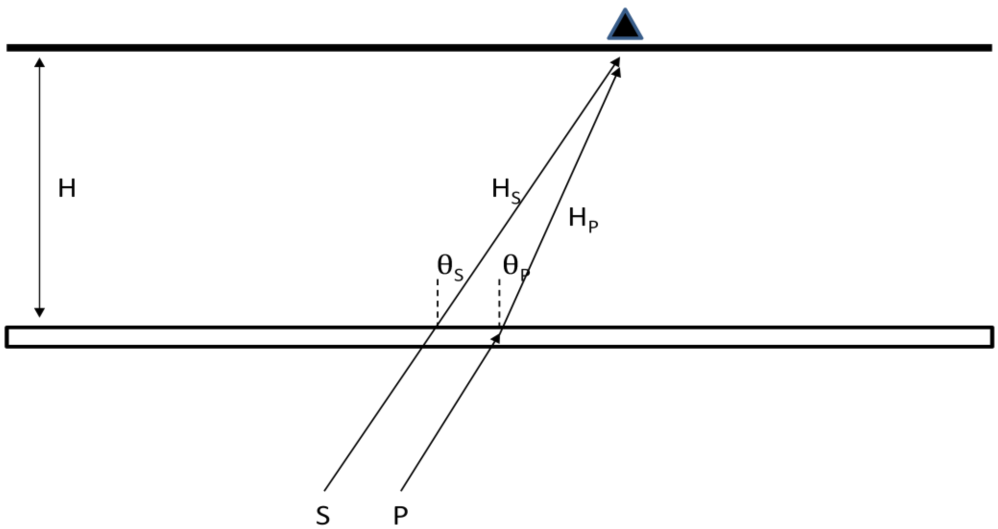
Attenuation can brighten S-wave receiver functions

Bill Menke, June 20, 2017

In this scenario, an incident S wave is scattered by a thin horizontal layer, generating a transmitted S wave and a converted P wave. The brightness of the S-wave receiver function depends of the relative amplitudes of the P and S wave, as recorded by the receiver.



By Snell's law:

$$\frac{\sin\theta_P}{v_P} = \frac{\sin\theta_S}{v_S} \quad \text{or} \quad \theta_P = \sin^{-1}\left(\frac{v_P}{v_S}\sin\theta_S\right)$$

The ray paths above the scattering layer are of length

$$H_S = \frac{H}{\cos\theta_S} \quad \text{and} \quad H_P = \frac{H}{\cos\theta_P}$$

The amplitudes of the waves are attenuated:

$$A_S = A_{S0}\exp\left(-\frac{\omega H_S}{2Q_S v_S}\right) \quad \text{and} \quad A_{P0} = \exp\left(-\frac{\omega H_P}{2Q_P v_P}\right)$$

Here A_{S0} and A_{P0} are the amplitudes of the wave just above the scattering layer, ω is angular frequency and Q_S and Q_P are quality factors. At the station, the amplitude ratio is:

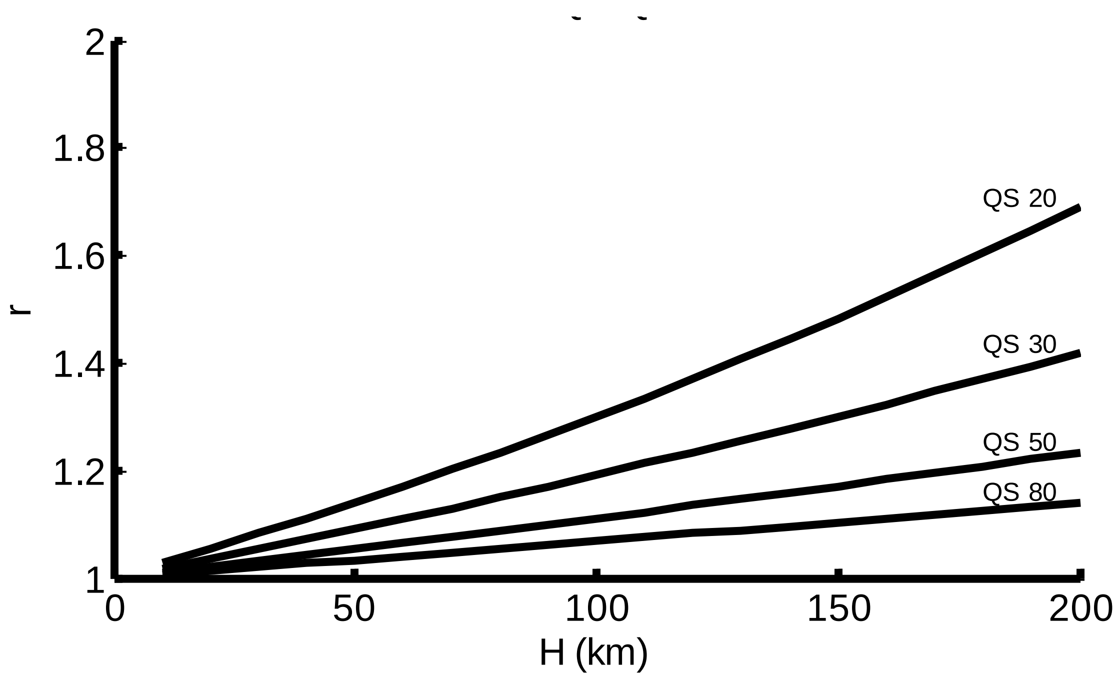
$$\frac{A_P}{A_S} = \frac{A_{P0}}{A_{S0}}\exp\left(-\frac{\omega}{2}\left[\frac{H_P}{Q_P v_P} - \frac{H_S}{Q_S v_S}\right]\right)$$

and the change in the ratio, relative to its original value, is:

$$r = \left(\frac{A_P}{A_S}\right) \Big/ \left(\frac{A_{P0}}{A_{S0}}\right) = \exp\left(-\frac{\omega}{2}\left[\frac{H_P}{Q_P v_P} - \frac{H_S}{Q_S v_S}\right]\right)$$

We perform exemplary calculations for:

variable	value
$f = \omega/(2\pi)$	0.1
H	$10 - 200$ km
v_S	4.5 km/s
v_P/v_S	1.8
Q_S	$20 - 80$
Q_P/Q_S	$9/4$
θ_S	$20°$



The results indicate that the attenuation increases the P to S amplitude ratio by as much as a factor of 70%, in the case of $H = 200$, $Q_S = 20$.