Sensitivity of the Seismic Velocity of Olivine to Perturbations in Magnesium Number

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Consider the mineral olivine,

$$(\mathrm{Mg}_{0.8}, \mathrm{Fe}_{0.2})_2\mathrm{SiO}_4$$

perturbed by changing the relative amounts of magnesium and iron via a parameter N (which represents the change in magnesium number):

$$\left(\mathrm{Mg}_{0.8+N/100}, \mathrm{Fe}_{0.2-N/100}\right)_2\mathrm{SiO}_4$$

The formula weight is:

$$W = 2(0.8 + N/100)M_{Mg} + 2(0.2 - N/100)M_{Fe} + M_{Si} + 4M_O$$

with individual elements having atomic weights:

$$M_{\mathrm{Mg}} = 24.304 \quad \text{and} \quad M_{\mathrm{Fe}} = 55.845 \quad \text{and} \quad M_{Si} = 28.084 \quad \text{and} \quad M_O = 15.999$$

The unperturbed weight is:

$$W_0 = 2(0.8)M_{Mg} + 2(0.2)M_{Fe} + M_{Si} + 4M_O = 153.301$$

Differentiating with respect to N yields:

$$\frac{\partial W}{\partial N} = \frac{2}{100}\left(M_{Mg} - M_{Fe}\right) = -0.631$$

The density ρ is the weight divided by the volume U_0, which is assumed constant (that is, it does *not* to vary with N). The fractional change in density ρ is:

$$\frac{\Delta\rho}{\rho_0} = \frac{\Delta W/U_0}{W_0/U_0} = \frac{\Delta W}{W_0} = \frac{1}{W_0}\frac{\partial W}{\partial N}\Delta N = \frac{-0.631}{153.301}\Delta N = -0.00411\Delta N$$

According to Menke (2016), at fixed values of the Lame constants, a perturbation in density leads to perturbations in compressional and shear velocity:

$$\Delta V_p = -½V_{p0}\frac{\Delta\rho}{\rho_0} = -½(8.30)(-0.00411)\Delta N = 0.0171\ \Delta N$$

$$\Delta V_s = -½V_{s0}\frac{\Delta\rho}{\rho_0} = -½(4.52)(-0.00411)\Delta N = 0.0093\ \Delta N$$

$$\frac{\Delta V_p}{\Delta V_s} = \frac{V_{0p}}{V_{0s}} = 1.836$$

The sensitivities inferred from Chung's (1970) experimentally determined velocities are somewhat different. Chung's (1970) laboratory measurements are:

N	V_p	ΔV_s
95	8.287	4.823
90	8.266	4.769
85	8.088	4.676
80	8.017	4.615

A least-squares regression of them yields:

$$\Delta V_p = 0.02166\,\Delta N \quad \text{and} \quad \Delta V_s = 0.01586 \quad \text{and} \quad \frac{\Delta V_p}{\Delta V_s} = 1.366$$

References:

Chung, D.H, Effects of Iron/Magnesium Ration on P- and S-Wave Velocities in Olivine, J. Geophys. Res. 75, 7353-7360, 1970.

Menke, W. Menke Research Note 164, Effect of Perturbation in Density on Vp/Vs Ratio, https://www.ldeo.columbia.edu/users/menke/research_notes/menke_research_note164.pdf