Harmonic Series for Products of Powers of Sines and Cosines

Bill Menke, December 4, 2019

I worked out these identities from first principles and have verified them numerically.

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$\sin\theta$

$\cos\theta$

$\sin^2\theta = ½ - ½\cos 2\theta$

$\sin\theta\cos\theta = ½\sin 2\theta$

$\cos^2\theta = ½ + ½\cos 2\theta$

$\sin^3\theta = (3/4)\sin\theta - ¼\sin 3\theta$

$\sin^2\theta\cos\theta = ¼\cos\theta - ¼\cos 3\theta$

$\sin\theta\cos^2\theta = ¼\sin\theta + ¼\sin 3\theta$

$\cos^3\theta = (3/4)\cos\theta + ¼\cos 3\theta$

$\sin^4\theta = (3/8) - ½\cos 2\theta + (1/8)\cos 4\theta$

$\sin\theta\cos^3\theta = ¼\sin 2\theta + (1/8)\sin 4\theta$

$\sin^2\theta\cos^2\theta = (1/8) - (1/8)\cos 4\theta$

$\sin^3\theta\cos\theta = ¼\sin 2\theta - (1/8)\sin 4\theta$

$\cos^4\theta = (3/8) + ½\cos 2\theta + (1/8)\cos 4\theta$

$\sin 2\theta \sin^2 \theta = ½ \sin 2\theta - ¼ \sin 4\theta$

$\sin 2\theta \cos^2 \theta = ½ \sin 2\theta + ¼ \sin 4\theta$

$\sin 2\theta \sin \theta \cos \theta = ¼ - ¼ \cos 4\theta$

$\cos 2\theta \sin^2 \theta = -¼ + ½ \cos 2\theta - ¼ \cos 4\theta$

$\cos 2\theta \cos^2 \theta = ¼ + ½ \cos 2\theta + ¼ \cos 4\theta$

$\cos 2\theta \sin \theta \cos \theta = ¼ \sin 4\theta$