Station Amplitude and Attenuation can be Uniquely Determined

Bill Menke with discussion with Andrew Lloyd, January 24, 2025

This proof considers the special case of two stations observing waves from opposite directions.

Consider the amplitude of a horizontally-propagating of a wave observed on two stations, 1 and 2. For left-to-right propagation the amplitude is given by the product of a reference amplitude A_0, station correction $(1 \pm S)$ and an attenuation $\exp(-q)$:

$$A_1 = A_0(1 - S) \quad \text{and} \quad A_2 = A_0(1 + S)\exp(-q)$$

Note that the station correction is defined so that the average amplitude is independent of it. Taking the logarithm and using $\ln(1 \pm S) \approx \pm S$ leads to equations E_1 and E_2

$$E_1\!: \; \ln A_1 = \ln A_0 - \mathrm{S} \quad \text{and} \quad E_2\!: \ln A_2 = \ln A_0 + S - q$$

For right-to-left propagation the log-amplitude is given by equations E_3 and E_4

$$E_3\!: \; \ln A_1' = \ln A_0' - \mathrm{S} - \mathrm{q} \quad \text{and} \quad E_4\!: \ln A_2' = \ln A_0' + S$$

In matrix form, E_1 through E_4 constitute four equations in four unknowns:

$$\begin{matrix} E_1: \\ E_2: \\ E_3: \\ E_4: \end{matrix} \quad \mathbf{d} \equiv \begin{bmatrix} \ln A_1 \\ \ln A_2 \\ \ln A_1' \\ \ln A_2' \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 1 & 0 & 1 & -1 \\ 0 & 1 & -1 & -1 \\ 0 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} \ln A_0 \\ \ln A_0' \\ S \\ q \end{bmatrix} \equiv \mathbf{Gm}$$

The matrix **G** can be upper-triangularized as follows:

$$\begin{matrix} E_1' = E_1: \\ E_2' = E_4: \\ E_3' = E_2 - E_1: \\ E_4' = E_4 - E_3 - E_3': \end{matrix} \quad \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 2 & -1 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

The matrix can be inverted, as the diagonal elements are non-zero. Consequently, the unknowns can be uniquely determined.