Vertical Transverse Isotropy of Olivine Under Upwelling Conditions

Bill Menke, July 27, 2026

These results are based on a Monte Carlo simulation implemented in Python.

I start with the orthorhombic olivine elastic tensor given by Fuchs (1983, 5th row in his table), rotated so that its fast axis is vertical. I consider a model where mantle upwelling aligns the fast axes of olivine grains so that they scatter about the vertical direction. The bulk elastic tensor is computed by averaging many (say, 10,000) tensors, each representing a mineral grain in an aggregate and generated by rotating the original tensor by a random amount around its fast axis and then tilting that axis by an angle drawn from a von Mises – Fisher pdf (Wikipedia, 2026a). In this pdf, the degree of alignment is controlled by the concentration parameter κ, which varies between $\kappa = 0$ (no alignment) to $\kappa = \infty$ (perfect alignment). I verified that this process generates a tensor that closely approximates that Vertical Transverse Isotropic ideal (Wikipedia, 2009b). In Voigt notation:

$$C_{22} = C_{11} \text{ and } C_{55} = C_{44} \text{ and } C_{12} = C_{11} - 2C_{66} \text{ and various other } Cs \text{ zero}$$

Shear wave velocities were calculated by solving the set of three 3×3 eigenvalue problems corresponding to waves propagating along each of the coordinate directions (Aki and Richards, 2009, their equation 5.106):

Parameters in the range $0 < \kappa \leq 5$ correspond to fast shear velocity v_{Sv} that increases with κ, slow shear velocity v_{Sv} that decreases with κ, anisotropy A that increases with κ from 0% to about 8%, and an approximately constant Voigt average velocity $v_{Voigt} \approx 4760 \ m/s$. Here:

$$v_{Voigt} = \sqrt{\frac{2v_{Sv}^2 + v_{Sh}^2}{2}} \quad \text{and} \quad A = \frac{v_{Sv} - v_{sh}}{v_{Voigt}}$$

Scatter plots of the fast axes using a Lambert equal area projection (Wikipedia, 2026c) illustrate that the higher values of the concentration parameter (e.g., $\kappa = 5$) correspond to only a modest degree of alignment.

Aki and Richards, Quantitative Seismology, Second Edition, University Science Books, 2009.

Fuchs, K, Phys. Earth Planet. Int. 31, 93-118, 1983.

Wikipedia, von Mises–Fisher distribution, last accessed July 27, 2026a, https://en.wikipedia.org/wiki/Von_Mises%E2%80%93Fisher_distribution

Wikipedia, Transverse isotropy, last accessed July 27, 2026a, https://en.wikipedia.org/wiki/Transverse_isotropy

Wikipedia, von Mises–Fisher distribution, last accessed July 27, 2026b, https://en.wikipedia.org/wiki/Lambert_azimuthal_equal-area_projection

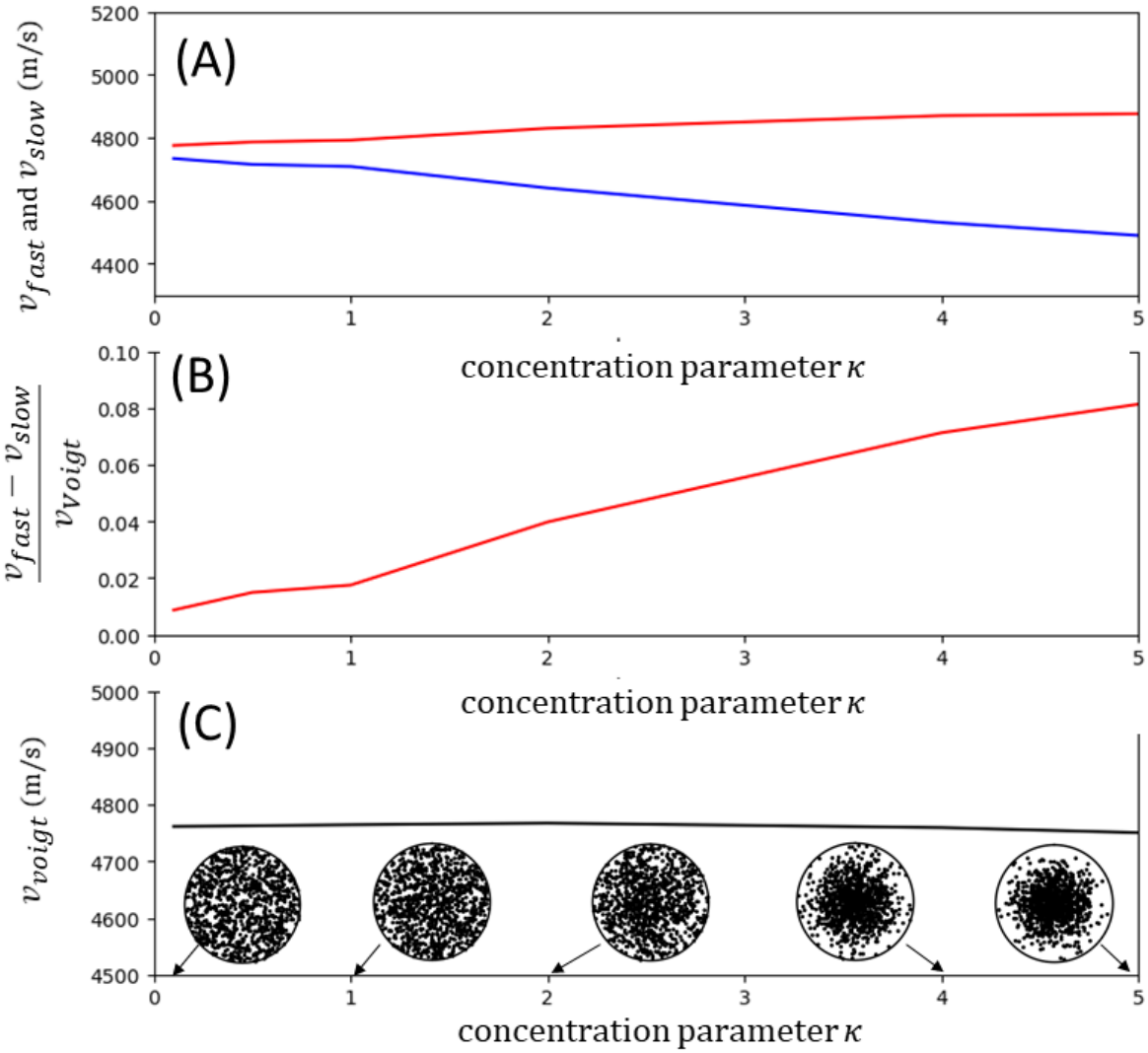

Fig. 1. Results of simulation. (A) Fast shear velocity v_{fast} (red) and slow shear velocity v_{slow} (blue) as a function of concentration parameter κ. (B) Anisotropy as a function of κ. (C) Voigt average velocity v_{fast} as a function of κ. Upper-hemisphere Lambert equal area scatter plots of fast axes for selected κs are inset.