The Pearson Correlation Coefficient is Preserved when Vectors Are Averaged

William Menke, September 17, 2026

Suppose that a model parameter vector $\mathbf{m}^{(i)}$, say of length M, is estimated for a single event i and that the elements of $\mathbf{m}^{(i)}$ have covariance matrix $\mathbf{C}_m$. The $\mathbf{m}$s estimated for different events are assumed to be uncorrelated, because the events are well-separated in time, and as the noise is assumed to be stationary, $\mathbf{C}_m$ is independent of i.

The average $\bar{\mathbf{m}}$ of, say, N model parameter vectors is defined as:

$$\bar{\mathbf{m}} \equiv \mathbf{M}\left[\mathbf{m}^{(1)}, \mathbf{m}^{(2)}, \cdots, \mathbf{m}^{(N)}\right]^T \quad \text{with} \quad \mathbf{M} \equiv N^{-1}[\mathbf{I}, \mathbf{I}, \cdots, \mathbf{I}]$$

The standard formula for error propagation then implies that the covariance of the average is:

$$\boldsymbol{C}_{\bar{m}} = \mathbf{M}\begin{bmatrix} \mathbf{C}_m & \mathbf{0} & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & \mathbf{0} & \mathbf{C}_m \end{bmatrix}\mathbf{M}^{\boldsymbol{T}} = N^{-1}\mathbf{C}_m$$

The covariance $\boldsymbol{C}_{\bar{m}}$ of the average is smaller than the covariance $\mathbf{C}_m$ of the individual model parameters, but the ratio of its elements are the same. The overall size of $\boldsymbol{C}_{\bar{m}}$ decreases with the number N of vectors that are averaged, following the same N^{-1} rule as in the better-known uncorrelated case. Consequently, the standard error of the jth average parameter $\sigma_j \equiv [\boldsymbol{C}_{\bar{m}}]_{\boldsymbol{jj}}^{1/2}$ is proportional to $N^{-1/2}$. The Pearson correlation coefficient $r_{jk} \equiv [\boldsymbol{C}_{\bar{m}}]_{jk}[\boldsymbol{C}_{\bar{m}}]_{jj}^{-1/2}[\boldsymbol{C}_{\bar{m}}]_{kk}^{-1/2}$ is independent of N and equal to the analogous coefficient for $\mathbf{C}_m$. In this sense, the averaging has preserved the intra-vector correlations.