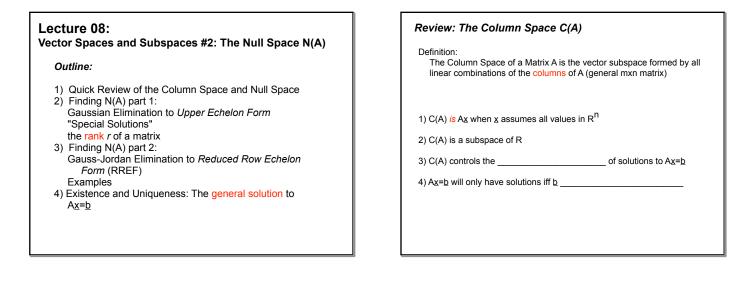
## Lecture08

### September 27, 2007



### Review: The Null Space N(A)

Definition:

The Null Space of a Matrix A is the vector subspace formed by all solutions  $\underline{x}$  to  $A\underline{x}=\underline{0}$  (i.e. is all linear combination of columns of A that cancel to zero)

1) N(A) is a subspace of R

2) N(A) controls the \_\_\_\_\_\_ of solutions to Ax=b

If A is invertible, then N(A)=\_\_\_\_\_

#### The Null Space: Algorithms for finding N(A)

1) Inspired Guessing A = [1 2; 3 6]

2) Gaussian Elimination to solve Ax=0

(Only small issue is that U is no longer upper triangular)

#### The Null Space:

Algorithms for finding N(A): Gaussian Elimination to Upper Echelon Form

A bigger example A = [1 1 2 1 ; 1 2 1 1]

Solve Ax=0 using Gaussian Elimination

#### Finding N(A): A closer look

Gaussian Elimination (including row exchanges) transforms a general mxn matrix A to U which is now "upper echelon form"

1) Identify "Pivot" columns and "Free columns" (and associated Pivot variables and free variables)

2) Identify the *rank* of the matrix r = number of Pivot Columns

3) Identify the number of special solutions = number of free columns =

4) Find the Special Solutions: the linear combinations of Pivot Columns required to annihilate a single free column.

5) The Null Space is formed by *all linear combinations* of the special solutions

## Lecture08

# September 27, 2007

example A = [1 1 2 1 ; 1 2 1 1 ] again

The Null Space: A better way Gauss-Jordan Elimination to Reduced Row Echelon Form R=rref(A)
Again A = [1 1 2 1 ; 1 2 1 1 ]
1) take A to U By Gaussian Elimination
2) Continue by GJ Elimination (eliminate up then divide by the pivots) to R $\!\!\!\!\!\!$
3) when done: The pivot columns will be columns of the Identity Matrix Can read the Special Solutions right out of R

#### The Null Space: A better way

Gauss-Jordan Elimination to Reduced Row Echelon Form R=rref(A)

Example A = [1 1 2 1 ; 1 2 1 1] goes to R = [ 1 0 3 1 ; 0 1 -1 0 ]

Point: N(R)=N()

But Rx=0 is much easier to see

!! The Special solutions are the linear combinations of the pivot columns that annihilate each free column.

#### The Null Space: A better way

Gauss-Jordan Elimination to Reduced Row Echelon Form R=rref(A)

Last Example: A = [ 1 2 2 5 ; 2 4 8 18 ; 3 6 10 23 ]

#### The Null Space: A Trick

Gauss-Jordan Elimination to Reduced Row Echelon Form R=rref(A)

Last Example: A = [ 1 2 2 5 ; 2 4 8 18 ; 3 6 10 23 ] R = [ 1 2 0 1 ; 0 0 1 2 ; 0 0 0 0 ]

Do it yourself Null Space:

#### Last-Last one (Easy)

Last Example: A = [ 1 2 ; 3 4 ]

R =

N(A)=

Point: All Invertible Matrices R=I, N(A)=Z

# Lecture08

# September 27, 2007

### Putting it all together: The General Solution to $A\underline{x}=\underline{b}$

Basic Approach:

1) use Gauss-Jordan Elimination to take [Ab] to [Rd]

2) Find a Particular solution to  $Rx_p = d$ (combination of pivot columns and no free columns that add up to <u>d</u>)

3) Find the special solutions to  $R\underline{x}_N = \underline{0}$ 

4) The General solution: is  $\underline{x} = \underline{x}_p + \underline{x}_N$ 

Putting it all together: The General Solution to Ax=b

Example: A = [ 1 2 1 0 1; 2 4 1 0 0 ; 1 2 0 1 -4 ] b = [ 1 1 1 ]

