Lecture 11:
Projections

Outline:

1) Quick Review of the Big Picture of the 4 subspaces

2) The picture for a Full Column rank matrix

3) Motivation for Projections (The small picture)

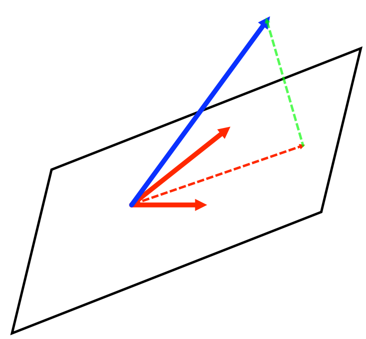
4) Projection onto a line

5) Projection onto a subspace and the "normal equations" $A^T A\hat{\underline{x}} = A^T \underline{b}$

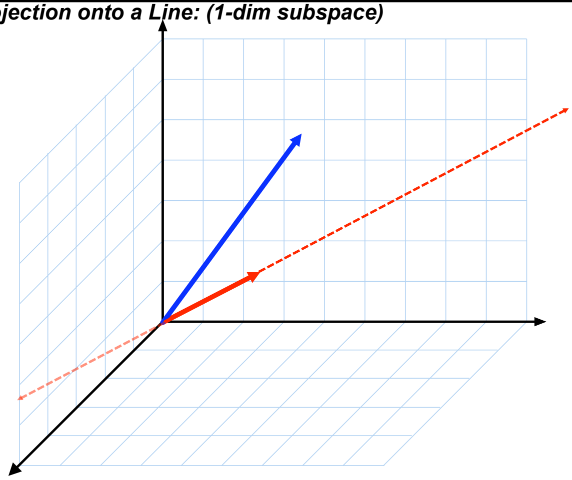
Orthogonality of the 4 subspaces:
The Big (Strangian) Picture

The Big Picture of $A\underline{x}=\underline{b}$
Full Column Rank:

The Small Picture of a Full Column rank problem:
Motivation for Projections and "Least-Squares Problems"



Projection onto a Line: (1-dim subspace)



Projection onto a Line: (1-dim subspace)

Steps:

1) *find the "least squares solution"* $\hat{x}$

2) *find the projection* p

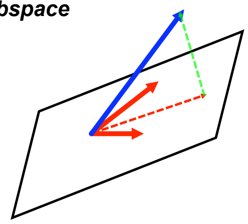
3) *find the Projection Matrix P*

Projection onto a Line: (1-dim subspace)

Example:

Project b=[1 1 1]' onto the line spanned by a=[1 2 2]'

Orthogonal Projection onto a Subspace



Orthogonal Projection onto a Subspace S

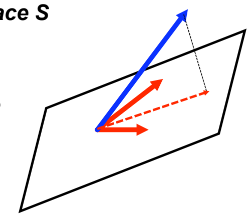
Steps:

0) Form a matrix whose columns span the subspace (i.e. S=C(A))

1) find the "least squares solution" $\hat{\underline{x}}$

2) find the projection $\underline{p}=A\hat{\underline{x}}$

3) find the Projection Matrix $P=A(A^TA^{-1})A^T$ (but don't use it really)



Projection onto a Subspace:

Example:

Project $\underline{b}$=[6 0 0]' onto the space spanned by $\underline{a}_1$=[1 1 1]' and $\underline{a}_2$=[0 1 2]'

Projection onto a Subspace:

Theorem:

if a matrix A is full-column rank, then A^TA is invertible

Proof:

1) Show that $N(A^TA)=N(A)$

2) if A is full Column Rank then $N(A)=N(A^TA)$=_________

Comment: If A is full-column rank then the linear least-squares solution is unique.

Projection onto a Subspace:

Theorem:

Even if A is not full column rank, the normal equations always have at least one solution.

Proof:

Show that A^Tb is always in $C(A^TA)$