Lecture 15:
Everything you wanted to know about the Determinant |A|

Outline:

1) Introducing the Determinant of a square matrix det(A)
2) Basic Properties: the big 3
3) More properties
4) Calculating |A|
   The Big Formula
   Calculation by Cofactors: Examples
5) Applications
   Cramer's Rule
   Volume and coordinate transformations
   Computing Eigenvalues

The Determinant of a square matrix A

Notation: |A| or det(A)

The determinant of a square matrix A is a function that maps the components of an nxn matrix to a number.

1) if A is singular, det(A)=0

2) the determinant of a 2x2 matrix

The Determinant of a square matrix A

Formal Definition: The determinant of a square matrix A is the unique alternating, multilinear function that maps a matrix to a scalar such that det(I)=1.

(this is not terribly informative though...)

In English (ala Strang): The Determinant has three fundamental properties

1) det(I)=1

2) Exchanging two rows in A changes the sign of |A|

Comment: if P is a permutation matrix |P| = ______

The Determinant of a square matrix A

The Determinant has three fundamental properties

1) det(I)=1
2) Exchanging two rows in A changes the sign of |A|
3) The determinant is linear by Rows

comments: 1) det(tA)=___________
2) these three properties are enough to uniquely define the determinant (and derive "the Big Formula)

The Determinant of a square matrix A

More Properties of the Determinant:

4) The determinant of a matrix with a repeated row is zero

The Determinant of a square matrix A

More Properties of the Determinant:

5) Elimination doesn't change the determinant

comment: if elimination takes A to U, |A| = +- |U|

The Determinant of a square matrix A

More Properties of the Determinant:

6) if A has a row of zeros, then det(A)=0

The Determinant of a square matrix A

More Properties of the Determinant:

7) If A is triangular: det(A) = $a_{11}a_{22}a33...a_{nn}$ (product of the diagonals)

8) if A is singular det(A)=0

The Determinant of a square matrix A

More Properties of the Determinant: (big one!)

9) Product rule: $|AB|=|A||B|$
Proof: let $D(A) = |AB|/|B|$ and show $D(A)$ satisfies the three fundamental rules

The Determinant of a square matrix A

More Properties of the Determinant:

10) Consequences of the product rule

A) $|A|=\pm|U|$ = product of the pivots

B) $|A^{-1}|=1/|A|$

C) $|A^T|=|A|$

D) $|Q|=\pm 1$

Computing det(A)

Method 1) $|A|=\pm|U|$ = product of the pivots (matlab)

Method 2): The Big formula (Liebniz formula)

Computing det(A)

Method 3) Computation by Cofactors

Computing det(A)

Computation by Cofactors: 3x3 matrix

Computing det(A)

Computation by Cofactors:
Example 1: A=[1 1 1 ; 0 2 2 ; 0 0 3]

Computing det(A)

Computation by Cofactors:
Example 2: A=[0 1 2 1 ; 0 4 0 1 ; 1 0 0 1; 0 1 1 1]

Applications of the determinant

Cramer's Rules: an old fashioned way to solve $A\underline{x}=\underline{b}$ (don't do this)