## Lecture 16:
**Introduction to eigensystems**

***Outline:***

1) Introduction: Eigenvalues, Eigenvectors: $A\underline{x}=\lambda\underline{x}$
2) An example
3) Motivation: The applications
   Iterative maps and matrix powers
   Dynamical Systems $d\underline{u}/dt=A\underline{u}$
4) An Algorithm for finding eigenvalues and eigenvectors
5) More Examples
6) simple checks Tr(A), |A|

## Introduction to Eigen Problems

***The course so far:***

***Part 1: $A\underline{x}=\underline{b}$ leads to $PA=LU$***

***Part 2: $A^TA\underline{x}=A^T\underline{b}$ leads to $A=QR$***

***Part 3: Fundamental Equation is $A\underline{x}=\lambda\underline{x}$***

***where A is square nxn***
***$\lambda$ is an Eigenvalue and***
***$\underline{x}$ is an Eigenvector***

***special directions such that $A\underline{x}$ behaves like scalar multiplication***

***(slightly misleading equation, we need to solve for both $\lambda$ and $\underline{x}$)***

***(Factorization is $A=S\Lambda S^{-1}$ or $A=Q\Lambda Q^T$)***

## Eigenvalues and Eigenvectors

***An Example: A=[ 1 2; 2 1]***

## Eigenvalues and Eigenvectors

***Foreshadowing: an Application -- iterative maps***

***A large number of numerical methods can be written as an iterative method***

$\underline{x}_{k+1}=A\underline{x}_k$ *i.e.*

## Eigenvalues and Eigenvectors

*An algorithm for finding Eigenvalues and Eigenvectors (of small matrices)*

*1) First find the eigenvalues*

## Eigenvalues and Eigenvectors

*An algorithm for finding Eigenvalues and Eigenvectors (of small matrices)*

*1) First find the eigenvalues*

*Example: A=[ 1 2 ; 2 1 ]*

## Eigenvalues and Eigenvectors

*An algorithm for finding Eigenvalues and Eigenvectors (of small matrices)*

*2) Find the eigenvectors as $\underline{x}_i = N(A-\lambda_i I)$*

*Example: A=[ 1 2 ; 2 1 ]*

## Eigenvalues and Eigenvectors

*An algorithm for finding Eigenvalues and Eigenvectors (of small matrices)*

*Some important Checks:*

*Example: A=[ 1 2 ; 2 1 ]*

## Eigenvalues and Eigenvectors

*An algorithm for finding Eigenvalues and Eigenvectors (of small matrices)*

*Example 2: Eigenvalues and Eigenvectors of a general 2x2 matrix*

## Eigenvalues and Eigenvectors

*An algorithm for finding Eigenvalues and Eigenvectors (of small matrices)*

*Example #3: 3x3 problem A = [ 1 1 1 ; 0 2 1; 0 0 3]*

## Eigenvalues and Eigenvectors

*An algorithm for finding Eigenvalues and Eigenvectors (of small matrices)*

*Some Cautions:*
*1) Elimination changes eigenvalues!*

*2) Eigenvalues can be complex!*

## Eigenvalues and Eigenvectors

*An algorithm for finding Eigenvalues and Eigenvectors (of small matrices)*

*Some Cautions:*
*3) Repeated Eigenvalues can lead to repeated eigenvectors (not linearly independent)...(but not always)*

*Example: A=[ 2 1 ; 0 2]*

## Eigenvalues and Eigenvectors

*Next step: Diagonalization and factorization* $A=S\Lambda S^{-1}$