Lecture 19:
Application of Diagonalization: Linear Dynamical systems and the dynamics of love affairs

Outline:

1) Linear Dynamical systems $d\underline{u}/dt=A\underline{u}$; $\underline{u}(0)=\underline{u}_0$
   Definitions and Interpretation
   General Solution and the matrix exponential e^A
   Interpretation of General Solution as a change of Basis

2) Examples: The Romeo and Juliet Problems
   The Reactive Model
   The Contrarian Model

3) General Classification of fixed points for 2x2 systems

Linear Dynamical systems:

Definition: an autonomous, linear dynamical system can be written as

$$d\underline{u}/dt = A\underline{u} \quad \underline{u}(0)=\underline{u}_0$$

where $\underline{u}$ is a state vector, $A\underline{u}$ is a vector that describes how $\underline{u}$ changes with time and $\underline{u}_0$ is the initial state at time t=0

Physical example: $\underline{u}$=[x y]' is the position of a particle
$A\underline{u}$ is the velocity of the particle
$\underline{u}_0$=[x0 y0]' is the initial position

Linear Dynamical systems:

Geometric interpretation: $\underline{u}(t)$ is a trajectory (parameterized curve) where $A\underline{u}$ is the vector tangent to the curve at any point.

Question: What if u is an eigenvector of A?

Linear Dynamical systems:

General Solution: If A is diagonalizable, all autonomous linear dynamical systems have a general solution that depend only on the eigenvalues and eigenvectors of A (and the initial condition).

Derivation of general Solution:

Linear Dynamical systems:

The matrix Exponential: e^A

For Diagonal Matrices:

General definition:

Check for diagonalizable A

Linear Dynamical systems:

Interpretation of General Solution as a Change of basis!

Examples: The R&J problems
-or-
The Dynamics of Love affairs

Generously cribbed from Steven Strogatz: Non-linear dynamical systems and chaos

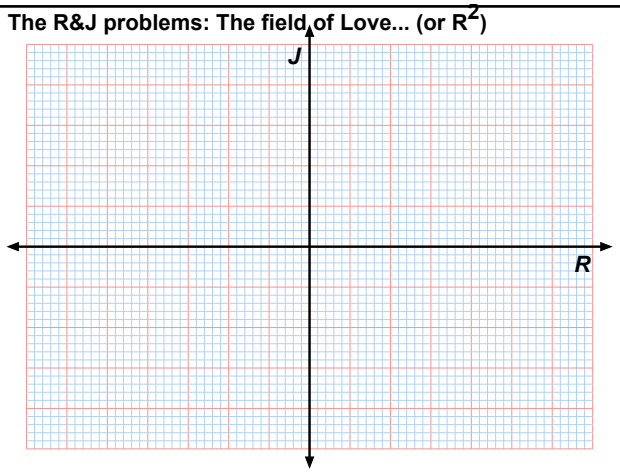
The Players: Romeo and Juliet

The Variables:

R: Romeo's love for Juliet
J: Juliet's love for Romeo

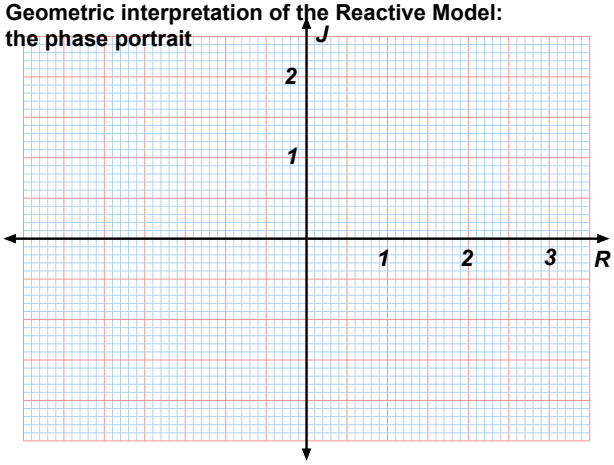
The Playing Field...

The R&J problems: The field of Love... (or R^2)



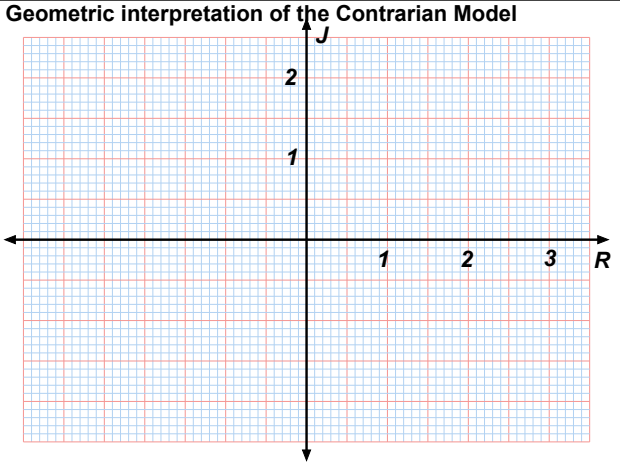
Examples #1: The Reactive Model

Geometric interpretation of the Reactive Model: the phase portrait



Examples #2: The Contrarian Model

Geometric interpretation of the Contrarian Model



Characterization of Fixed Points:

All 2x2 linear dynamical systems can be categorized by their eigenvalues

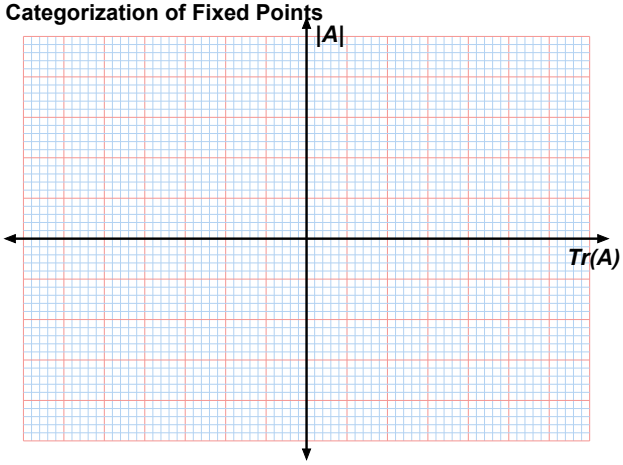
1) If A has real coefficients: the characteristic polynomial has either 2 real roots or a pair of complex conjugates

A) real eigenvalues can be positive, negative or zero (exponential growth or decay)

B) complex conjugates can be growing and decaying spirals or orbits

2) Behavior of Eigenvalues entirely controlled by Tr(A) and |A|

Categorization of Fixed Points



Introduction to non-linear dynamical systems