Heat Flow

Volcanos, intrusions, earthquakes, mountain building and metamorphism are controlled by transfer and generation of heat. The Earth's thermal budget controls the activity of lithosphere and asthenosphere as well as the development of innermost structure of the Earth.

At the Earth's surface heat arrives from the Earth's interior and from the Sun. The Sun and the biosphere have kept the surface temperature within the range of the stability of liquid water, probably $15 - 25°$C averaged over geological time. Given that constraint, the movement of heat derived from the interior has governed the geological evolution of the Earth, controlling plate tectonics, igneous activity, metamorphism, the evolution of the core and, hence, the Earth's magnetic field.

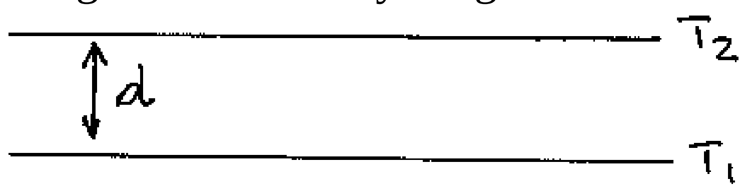
Heat moves by conduction, convection, radiation and advection.

- **Conduction** is the transfer of heat through a material by atomic or molecular interaction with the material.
- In **convection**, heat transfer occurs because the molecules themselves are able to move from one location to another within the material; convection is important in liquids and gases.
- **Radiation** involves direct heat transfer by electromagnetic radiadion (light, micro-waves like in a microwave oven)
- **Advection** is a special form of convection. If a hot region in

the Earth is uplifted by tectonic events or erosion and isostatic rebound, the heat (called advected heat) is physically lifted up with rocks. Subduction of a cold lithospheric plate, like in a subduction zone, is another example of advection.

Conductive heat flow

Imagine an infinitely long and wide solid plate, d in thickness:



The rate of heat per unit area through the plate is proportional to:

$$\frac{T_2 - T_1}{d}$$

The rate of heat per unit area up through the plate is therefore:

$$Q = -k\frac{T_2 - T_1}{d};$$

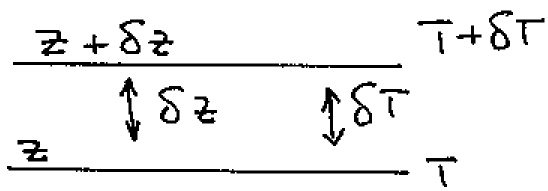
where k, the constant of proportionality, is called thermal conductivity. The rate of flow of heat per unit area is measured in units: watts per square meter (Wm^{-2}; thermal conductivity is measured in watts per meter per degree centigrade ($Wm^{-1}\,{}^{\circ}C^{-1}$). Examples of thermal conductivity:

silver: $k = 418 Wm^{-1}\,{}^{\circ}C^{-1}$

glass : $k = 1.2$

rocks : $k = 1.7 - 3.3$

wood : $k = 0.1$



$$Q_z = -k\left(\frac{(T+\delta T)-T}{(z+\delta z)-z}\right) = -k\frac{\delta T}{\delta z};$$

Following other considerations, it follows that if the rate of heat changes as a function of depth, there will be a change of temperature with time. Also, internal heat sources must be considered:

$$\frac{\delta T}{\delta t} = \frac{k}{\rho c_p}\frac{\delta}{\delta z}\left(\frac{\delta t}{\delta z}\right) + \frac{A}{\rho c_p}$$

where:

ρ – density

c_p – specific heat: amount of heat necessary to increase temperature of 1 kg by $1°$C

$\kappa = \frac{k}{\rho c_p}$ is thermal diffusivity

The equation above is called **heat conduction equation**; if $a = 0$ then:

$$\frac{\delta T}{\delta t} = \kappa\frac{\delta}{\delta z}\left(\frac{\delta T}{\delta z}\right)$$

is called the **diffusion equation**.

Example 1

Periodic variation of surface temperature:

$T(0,t) = T_0 \cos \omega t$

$T(z,t) \Rightarrow 0$ as $z \Rightarrow \infty$

$T(z,t) = T_0 \exp(-\sqrt{\frac{\omega}{2\kappa}} z) \cdot \cos(\omega t - \sqrt{\frac{\omega}{2\kappa}} z)$

where the exponentiql function $\exp(x) = e^x; e = 2.71828...$

At a depth of $L = \sqrt{2\kappa/\omega}$ the periodic disturbance has an amplitude of $1/e$ of the amplitude at the surface. This depth L is called *skin depth*. Taking $\kappa = 10^{-6} m^2 s^{-1}$, then the:

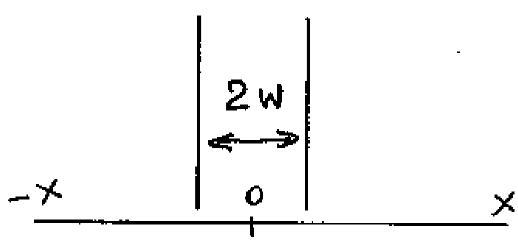
daily variation ($\omega = 7.27 \times 10^{-5} s^{-1}$) $L = 17$cm;

annual variation ($\omega = 2 \times 10^{-7} s^{-1}$) $L =$ 3.3 m

glaciation (100,000 years; $\omega = 2 \times 10^{-12} s^{-1}$) $L \approx 1$km.

Instantaneous cooling and heating

Let magma of temperature t_0 be injected into a dyke of width $2w$:



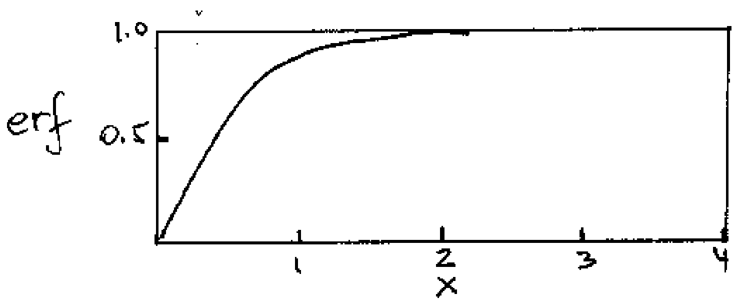
$T = T_0$ at $t = 0$ for $-w \leq x \leq w$ and

$T = 0$ at $t = 0$ for $\mid x \mid$

then

$T(x,t) = \frac{T_0}{2}\left[erf\left(\frac{w-x}{2\sqrt{\kappa t}}\right) + erf\left(\frac{w+x}{2\sqrt{\kappa t}}\right)\right]$

$erf = \frac{2}{\sqrt{\pi}} \int_0^x e^{-y^2}\, dy$



If the dyke were 2 m in width and intruded at a temperature of $1000°$C, $\kappa = 10^{-6} m^2 s{-}1$, then the temperature at the centre of the dyke would be $640°$C in one week, $340°$C after one month and only $100°$C after 1 year.

For the general case, the temperature in the dyke is about $T_0/2$ when $t = w^2/\kappa$ and about $T_0/4$ when $t = 5w^2/\kappa$.

High temperatures outside the dyke are confined to narrow contact zone: at a distance w away from the edge of the dyke the highest temperature reached is only about $T_0/4$. Temperatures close to $T^0/2$ are reached only within about $w/4$ of the edge of the dyke.

If we model a major intrusion, $w = 1$km and the initial temperature $T = 1000°$C:

z , t	10^4 years	10^6 years	10^8 years
10 km	0°C	50°C	10°C
100 km	0°C	0°C	5°C