Spatial and temporal variability of turbulent mixing in an estuary

by Hartmut Peters¹

ABSTRACT

Observations with a microstructure/CTD profiler and an ADCP during five cruises in 1994/95 provide a glance at the turbulent mixing climate of a partially mixed estuary, the Hudson River. The cruises took place under widely varying conditions of river discharge and external forcing by water level fluctuations in New York Bight. Tidal and fortnightly variability patterns of stratification, shear and mixing characteristics were qualitatively repeatable, largely following the description of Peters (1997). (i) Longer-term variations of buoyancy frequency and Richardson number \( R_i \) during flood were weakly correlated with the river discharge. (ii) All flood tides had substantial mixing. Cruise-to-cruise variations of turbulent dissipation rate, salt flux, and stress were large and correlated with variations in \( R_i \). (iii) During ebb tides, mixing was weak in response to stable stratification during neaps, and it increased dramatically toward spring tides parallel with the occurrence of low \( R_i \). Cruise-to-cruise variations of mixing became small toward spring tide. (iv) The largest vertical salt flux occurred during spring ebbs, which provided about 30% of the total fortnightly salt flux; floods throughout the fortnightly cycle providing most of the remainder. (v) The observed “level” of small-scale mixing, i.e. stress and vertical salt flux, was consistent with the integral momentum and salt budgets in the uniform segment of the Hudson where the bulk of the measurements were taken. (vi) One topographically forced flow feature examined, lee waves behind a trench across the river, was associated with elevated mixing.

1. Introduction

In the classical view of estuarine circulation, the freshwater river discharge and the lower layer saltwater influx from the ocean combine in a brackish, upper layer outflow to the ocean which can greatly exceed the freshwater inflow (e.g. Pritchard, 1952, 1954, 1956;

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Rattray and Hansen, 1962; Cameron and Pritchard, 1963; Hansen and Rattray, 1965; Pritchard, 1967). The conversion of saltwater and freshwater into brackish water is an irreversible process in thermodynamic terms, which can be achieved only by mixing and diffusion. Hence, turbulent mixing is essential in the estuarine circulation to lowest order, a statement that holds far beyond the narrow assumptions which underlie the classical estuarine theories. Following Geyer et al. (2000), detail of mixing in the water column is not important for the estuarine, or residual, circulation, however. This substantial simplification of the dynamics results from the fact that mixing even well above the bottom is tied to the bottom shear stress, a topic also discussed by Peters and Bokhorst (2000b). Nevertheless, systematic observations of turbulent mixing are necessary for understanding the estuarine flow and modeling it. Most estuaries are geometrically complex, and hydraulic processes and secondary circulations tied to bathymetric features are part of their dynamics (Stommel and Farmer, 1952; Chant and Wilson, 1997; Seim and Gregg, 1997). This raises the question whether the necessary mixing of freshwater and saltwater is localized; e.g., near hydraulic control points, or whether mixing is distributed more homogeneously over large stretches of estuaries.

Obviously, the answer to this question is different for different estuaries. In the fjord-like Puget Sound, e.g., mixing is weak in the deep and wide basins and intense in passages and over sills in response to hydraulic processes and secondary spanwise circulations (Seim and Gregg, 1994, 1995, 1997). In contrast, the observations from the Hudson River analyzed herein were based on the hypothesis that the classical estuarine circulation exists and that mixing across the halocline takes place over broad areas along and across the estuary rather than being dominated by localized processes. Accordingly, the observations were mostly taken in a bathymetrically rather uniform segment of the Hudson River with fairly uniform flow.

This paper examines the tidal, fortnightly and longer-term variability of turbulent mixing within the uniform segment of the estuary, relating these variations to the forcing of the estuary by the tides, the river discharge and other factors. The observed mixing is related to examinations of the integral momentum and salt budgets by Geyer et al. (2000). In addition, existing limited observations of the spatial variability of mixing related to topographic forcing are examined. This paper continues previous analyses in Peters (1997), henceforth referred to as P97, Peters and Bokhorst (2000a) and Peters and Bokhorst (2000b), henceforth called PB00a and PB00b, respectively. The analysis is based on five microstructure cruises in 1994/95 (see Section 2). The longer-term, fortnightly, and tidal variability of flow and mixing is analyzed in Section 3, and spatial variations are addressed in Sections 4 and 5. The paper concludes with summary and discussion.

2. Observations

a. Instrumentation

The core of the instrumentation is the microstructure profiler “SWAMP” (Shallow Water Microstructure Profiler; P97) equipped with dual shear probes measuring microscale
shear (Osborn and Crawford, 1980), a fast FP07 thermistor measuring temperature and temperature gradient, a Sea-Bird microstructure conductivity cell (during cruises HUDM and HUDM2—see Table 1) measuring electrical conductivity and conductivity gradient, regular Sea-Bird CTD sensors, and auxiliary tilt meters and accelerometer. The instrument is loosely tethered with a 6 mm Kevlar cable that transmits the data to the ship. SWAMP has its own dedicated winch that enables taking profiles as fast as one per minute in shallow water. During and after HUD3, we reduced the frequency of profiling because the cable was wearing out too fast.

The ambient horizontal and vertical currents and shear were measured with a shipborne 600-kHz broadband acoustic Doppler current profiler (ADCP) typically averaging 8 pings in 13 s. The ADCP was running in “mode 2” (see the Broadband ADCP manual of RD Instruments) during cruises HUD1-HUD2, and nominally in mode 4 thereafter. In reality, the ADCP was operating in mode 1 part of the time as concluded from an analysis of noise levels in PB00a. The switching between modes 1 and 4 was undocumented and not recorded by the manufacturer’s data acquisition software so that the actual operating mode at any one time is unknown. Noise in the ADCP velocity data is less important for Section 3, which is based on substantially averaged data, than in Sections 4 and 5, where less or no averaging is employed.

The combined SWAMP/ADCP data allow a relatively complete characterization of local fluid columns with respect to stratification, velocity, shear and gradient Richardson numbers, and to mixing parameters such as the viscous dissipation rate, $\varepsilon$, and overturning scales (Thorpe, 1977). While stratification and turbulence parameters stem from microstructure profiles, velocity and shear are from the ADCP. Richardson numbers combine ADCP shear and profiler buoyancy frequency. Section 3 contains a discussion of how eddy viscosity and eddy diffusivity, and turbulent fluxes of momentum and mass can be estimated from the measurements.

### b. Locale and cruises

Profiling microstructure observations were taken during five cruises in the Hudson River Estuary in 1994/95 as listed in Table 1. On each day, measurements spanned one semidiurnal tidal cycle or part thereof. Cruise HUD1 was analyzed in P97, while cruises

<table>
<thead>
<tr>
<th>Cruise</th>
<th>Dates</th>
<th>Year days</th>
<th>No. drops</th>
</tr>
</thead>
<tbody>
<tr>
<td>HUD1</td>
<td>May 19–20, 23–24, 1994</td>
<td>139–140, 143–144</td>
<td>1073</td>
</tr>
<tr>
<td>HUD2</td>
<td>Aug. 31, Sep. 1–2, 8–9, 1994</td>
<td>243–245, 251–252</td>
<td>1532</td>
</tr>
<tr>
<td>HUD3</td>
<td>April 8–14, 1995</td>
<td>98–104</td>
<td>1341</td>
</tr>
<tr>
<td>HUDM2</td>
<td>Oct. 18–23, 1995</td>
<td>291–296</td>
<td>1095</td>
</tr>
</tbody>
</table>

Table 1. Cruises in the Hudson River. Year day 1 is Jan. 1. “No. drops” is the number of good microstructure profiles.
HUDM and HUDM2 were discussed in PB00a and PB00b. Data from HUD2 and HUD3 have not been analyzed before. Measurements were taken from the anchored R/V *Onrust* during HUD1, and from the ship drifting with the tidal current thereafter, the bulk of the microstructure drops being

Figure 1. Map of the Hudson River Estuary with inset showing the locations of microstructure drops in rotated Cartesian coordinates indicated. For reference, the George Washington Bridge is at $y \approx 12$ km, and 59th Street is at $y = 2$ km.

HUDM and HUDM2 were discussed in PB00a and PB00b. Data from HUD2 and HUD3 have not been analyzed before. Measurements were taken off Manhattan Island, New York City (Fig. 1). Characterizations of the estuary and its circulation are provided in Section 3. Most figures in this section use the same key for origin by cruise shown in Figure 2.

The bulk of the microstructure profiles (or “drops”) were taken in a geometrically relatively uniform stretch of the river with a typical depth of 15 m, while one streamwise section extended further north beyond the George Washington Bridge through more variable bathymetry (Figs. 1 and 18). The Cartesian coordinate system used herein has its origin at 40.75N, 74.0W, and is rotated 30° clockwise such that $x$ is the spanwise axis and $y$ is the streamwise axis. The corresponding horizontal velocity vector is $\mathbf{u} = (u, v)$.

Measurements were taken from the anchored R/V *Onrust* during HUD1, and from the ship drifting with the tidal current thereafter, the bulk of the microstructure drops being
taken within $3 \text{ km} \leq y \leq 7.5 \text{ km}$ (Fig. 1). The sampling from the drifting ship mixed spatial and temporal variations during cruises HUD2-HUDM2. However, on semidiurnal and larger time scales, the variability was essentially temporal rather than spatial (PB00a). It appears that the peculiar sampling dictated by the technical demands of microstructure profiling had one advantage over fixed point measurements in providing averaging along and across a 2–4 km stretch of the estuary. Navigation used the Global Positioning System (GPS), differential GPS not being available. Hence, positions can be in error by as much as ±100 m.

c. **Data reduction**

Data reduction and data quality of Hudson microstructure observations have been discussed in P97 and PB00a and references therein. A special problem arose with the HUD2 microstructure velocity data, when dissipation rates much higher than during HUD1 occurred. The high $\varepsilon$ caused “railing” of the shear probe signals, voltages beyond the $\pm 5 \text{ V}$ range of the analog-to-digital converter that were clipped. Unfortunately, this problem could not be remedied during the cruise. The railing introduces a low bias in $\varepsilon$ of up to a factor of 5 in ensemble averages, and up to a factor of 50 in instantaneous data. We later reduced the amplification in the shear probe channels by a factor of 50 by modifying the electronics circuitry, and used programmable gain stages to boost signals during periods of low $\varepsilon$.

In order to save the observations from HUD2, a simple correction of the low bias in $\varepsilon$ introduced by the railing was devised. The scheme is entirely empirical, based on simulating the problems of the HUD2 shear probe data with their counterparts from HUDM, which do not suffer from railing. The HUDM data were used because conditions of river flow and temperature were similar between HUDM and HUD2. The shear probe voltages of HUDM were boosted with HUD2 amplification factors and then clipped at $\pm 5 \text{ V}$. Original and clipped data were run through the same standard routines of determining $\varepsilon$. The low bias of the clipped data is removed to an accuracy of better than 10% by the correction factor

$$c_{cc} = 10^{7.7(n_5 - 1.1n_5^2)},$$  

(1)
where \( n_5 \) is the fraction of data points at \( \pm 5 \) V, a variable ranging from 0 to 0.35. While the overall bias introduced by railing can be removed, the corrected \( \varepsilon_{cc} \) shows an increase in random scatter compared to the original \( \varepsilon \) (Fig. 3). The scatter in ensemble averages has a standard deviation equivalent to a factor of 2.4.

3. Tidal, fortnightly and longer-term variations

a. The estuary and its salt intrusion

The Hudson River enters New York Harbor at the Battery on the southern tip of Manhattan Island, river mile 0 (Fig. 1). New York Harbor is characterized by complex bathymetry and channels connecting it with Long Island Sound (East River) and with Newark Bay and Raritan Bay along the backside of Staten Island (Kill van Kull and Arthur Kill).\(^2\) Today, tides in the Hudson reach 245 km upstream from the Battery, to a dam at

\[\text{Figure 3. Corrected dissipation rates from shear probe signals clipped at } \pm 5 \text{ V, } \varepsilon_{cc}, \text{ versus original data free of railing, } \varepsilon; \text{ ensemble averages from HUDM treated to simulate problems with data from HUD2. The solid line is the median of } \varepsilon_{cc} \text{ in bins of } \varepsilon.\]
Green Island near Albany, NY, the port of Albany being accessible to oceangoing ships. Abood (1977) describes variations in the extent of the salt intrusion as function of river flow. At the extremes, salinity, $S$, drops below 2 practical salinity units (psu) at 30–130 km from the Battery. More typically, vertically averaged salinities of 1 psu extend into or beyond Haverstraw Bay, i.e. at least 60 km above the Battery.

The core of the observations analyzed herein were taken between roughly 10 and 16 km upstream of the Battery, i.e. well within the salt intrusion. As seen in Figure 1, this reach of the river is rather straight. Its cross section and depth are fairly uniform (Abood, 1977; Trowbridge et al., 1999). A typical depth of about 15 m is partly a result of ongoing dredging. Within the area investigated here, the longitudinal salinity gradient $\partial S/\partial y$ varied little as function of $y$, especially during spring tides (Geyer et al., 2000; Bowman, 1977). These characteristics place the observations analyzed into the “central regime” of the estuarine salinity distribution following Hansen and Rattray (1965).

b. Tides

Before discussing the observed river flow and its effects on the salinity distribution, it is necessary to introduce the tides, the dominant stirring agent creating turbulent mixing. Table 2 shows that tidal currents were at least one order of magnitude larger than the residual flow forced by river discharge and remote atmospheric forcing. Tidal stirring is nonlinearly related to tidal amplitudes (e.g. P97, PB00a), and thus seemingly small variations in tidal amplitude can cause large variations in mixing. The fortnightly tidal cycle is especially important in this context. Haas (1977) and Jay and Smith (1990) discuss the fortnightly modulation of flow and stratification, while P97 analyzes the corresponding variation of turbulent mixing. The semidiurnal amplitude of the water level at The Battery on the southern tip of Manhattan during the five cruises is depicted in Figure 4. This

<table>
<thead>
<tr>
<th>Cruise</th>
<th>Year day</th>
<th>Category</th>
<th>$v_a$</th>
<th>$\bar{v}$</th>
<th>$v_r$</th>
<th>$v_e$</th>
<th>$S$</th>
</tr>
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<tbody>
<tr>
<td>H111</td>
<td>199–201</td>
<td>neaps</td>
<td>0.73</td>
<td>−0.08</td>
<td>−0.04</td>
<td>0.11</td>
<td>8.7</td>
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<tr>
<td>H112</td>
<td>202–203</td>
<td>springs</td>
<td>1.14</td>
<td>0.05</td>
<td>−0.02</td>
<td>0.05</td>
<td>18.5</td>
</tr>
<tr>
<td>H113</td>
<td>204–205</td>
<td>neaps</td>
<td>0.75</td>
<td>0.09</td>
<td>−0.02</td>
<td>0.11</td>
<td>18.5</td>
</tr>
<tr>
<td>H114</td>
<td>206–207</td>
<td>springs</td>
<td>1.10</td>
<td>−0.05</td>
<td>−0.03</td>
<td>0.12</td>
<td>13.9</td>
</tr>
<tr>
<td>H115</td>
<td>208–209</td>
<td>neaps</td>
<td>0.97</td>
<td>−0.03</td>
<td>−0.05</td>
<td>0.15</td>
<td>16.8</td>
</tr>
<tr>
<td>H116</td>
<td>210–211</td>
<td>springs</td>
<td>1.10</td>
<td>−0.05</td>
<td>−0.03</td>
<td>0.12</td>
<td>13.9</td>
</tr>
<tr>
<td>H117</td>
<td>212–213</td>
<td>neaps</td>
<td>0.75</td>
<td>0.09</td>
<td>−0.02</td>
<td>0.11</td>
<td>18.5</td>
</tr>
<tr>
<td>H118</td>
<td>214–215</td>
<td>springs</td>
<td>1.14</td>
<td>0.05</td>
<td>−0.02</td>
<td>0.05</td>
<td>18.5</td>
</tr>
<tr>
<td>H119</td>
<td>216–217</td>
<td>neaps</td>
<td>0.73</td>
<td>−0.08</td>
<td>−0.04</td>
<td>0.29</td>
<td>18.7</td>
</tr>
<tr>
<td>H120</td>
<td>218–219</td>
<td>springs</td>
<td>1.03</td>
<td>−0.08</td>
<td>−0.05</td>
<td>0.10</td>
<td>18.0</td>
</tr>
<tr>
<td>H121</td>
<td>220–221</td>
<td>neaps</td>
<td>0.79</td>
<td>0.13</td>
<td>−0.01</td>
<td>0.13</td>
<td>21.9</td>
</tr>
<tr>
<td>H122</td>
<td>222–223</td>
<td>springs</td>
<td>1.01</td>
<td>−0.03</td>
<td>−0.01</td>
<td>0.08</td>
<td>22.5</td>
</tr>
<tr>
<td>H123</td>
<td>224–225</td>
<td>neaps</td>
<td>0.67</td>
<td>0.11</td>
<td>−0.02</td>
<td>0.06</td>
<td>22.2</td>
</tr>
<tr>
<td>H124</td>
<td>226–227</td>
<td>springs</td>
<td>0.81</td>
<td>0.21</td>
<td>−0.02</td>
<td>0.04</td>
<td>23.2</td>
</tr>
<tr>
<td>H125</td>
<td>228–229</td>
<td>neaps</td>
<td>1.00</td>
<td>−0.18</td>
<td>−0.12</td>
<td>0.11</td>
<td>16.2</td>
</tr>
</tbody>
</table>

Table 2. Characterization of days of observations within the fortnightly cycle. Shipboard ADCP velocity data: semidiurnal current amplitude ($v_a$ [m s$^{-1}$]), depth/time-averaged streamwise current ($\bar{v}$ [m s$^{-1}$]), average current resulting from river flow ($v_r$ [m s$^{-1}$]), rms current deviation from vertical mean ($v_e$ [m s$^{-1}$]), and depth-averaged salinity ($\bar{S}$ [psu]) from SWAMP profiles.
Figure 4. Semidiurnal tidal amplitude (thick solid line) and low-passed water level (shaded) from the Battery, New York City; river discharge at Green Island, NY (stair step line). Times of microstructure drops are also indicated. Year days are labeled at their beginning, 0000 universal time (UT).

The amplitude encompasses both M2 and S2 tides; it was determined by complex demodulation as described in PB00a employing a filter wide enough to pass both M2 and S2 signals.

All five cruises covered neap tides as well as spring tides to varying degrees. For the purpose of comparison within the fortnightly tidal cycle, individual days of observations
are classified as “neaps,” “transition,” and “springs” (compare Table 2 with Fig. 4). With highly variable placement of observations relative to the fortnightly cycle, the classification has to be taken with a grain of salt. P97 labeled year days 139–140 of HUD1 as neaps, but the fortnightly increase in tidal amplitude was well underway on these days, and thus classification as transition appears more appropriate in the context of this paper.

The semidiurnal amplitude of depth-averaged currents measured with the shipboard ADCP is listed in Table 2 as \( v_a \). The largest amplitudes at or above 1.1 m s\(^{-1}\) occurred during HUD2 and HUD1, while \( v_a \) reached only about 1.0 m s\(^{-1}\) during the other cruises. Compared to the remainder of the observations, HUDM had a small neap-spring increase in \( v_a \). PB00a note that the semidiurnal current amplitudes \( v_a \) observed from the Onrust was highly correlated with the corresponding amplitudes of the water level at the Battery.

Figure 5 shows depth-averaged velocities whose semidiurnal amplitude and residual are listed in Table 2. The velocities are displayed as a function of the semidiurnal tidal phase \( \Phi_{SD} \) based on the water level at the Battery introduced above. Time \( t \) assigns phase \( \Phi_{SD} \) to velocity \( v(t) \) as well as other variables. Note that the ebb sector corresponds to \( 90^\circ < \Phi_{SD} < 270^\circ \), while the flood sector is split into \( 0^\circ \leq \Phi_{SD} < 90^\circ \) and \( 270^\circ < \Phi_{SD} \leq 360^\circ \). Ensemble averages employed throughout this section are formed in 30° bins of \( \Phi_{SD} \) (45° bins in the “transition” data from HUDM2). Individual profiles were taken at substantially varying water depth \( H \) in relationship to varying profile location (Fig. 1) and tidal water level fluctuations (Fig. 4), and thus vertical averaging needs special consideration. In the vertical, averages are either formed over the entire water depth or in bins of normalized depth \( z/H \). Here, \( z \) is height above bottom and \( H \) is the actual water depth encountered at the time of measurement. A bin size of 0.15 in \( z/H \) corresponds to 2.4 m with an average depth of 16 m. Throughout, average vertical profiles are not displayed as a function of \( z/H \) but as a function of nominal dimensional height, \( 16 m \times z/H \), in order to provide a better sense of vertical proportion. Velocity measurements do not cover the top and bottom 2–2.5 m, stratification does not cover the top and bottom ~1 m, and dissipation rates omit the top 2.5–3.5 m. Dissipation data reach to within 15 cm of the bottom in most profiles from cruise HUD3-HUDM2. In earlier cruises the coverages of the near-bottom zone is irregular.

c. River flow

Naturally, the river flow greatly affects the estuarine circulation in more ways than just in the length of the salt intrusion. Climatologically, the Hudson has maximum discharge in spring and a weak secondary peak in early winter. The years 1994–95 partly deviated from this scheme, however (Fig. 6). Cruise HUD1 took place toward the end of the strong spring run-off of 1994, the river discharge dropping from 700 m\(^3\) s\(^{-1}\) to 300 m\(^3\) s\(^{-1}\) during the cruise (Fig. 4). The following year had no clear spring maximum run-off, and HUD3 had discharge rates of around 500 m\(^3\) s\(^{-1}\), similar to HUD1. HUD2 and HUDM both took place during ordinarily low summer river flow, HUD2 having insignificantly larger than average discharge of 200–300 m\(^3\) s\(^{-1}\) while HUDM was affected by the drought of 1995 with small river flow of only about 100 m\(^3\) s\(^{-1}\). By the time of cruise HUDM2, precipitation had
increased with resulting increased discharge rates of 200–300 m$^3$/s$^{-1}$. A major storm with torrential downpour on year day 294 elevated the river flow to 1400 m$^3$/s$^{-1}$. The river discharge is monitored at Green Island, 245 km upstream from the Battery. Variations in discharge propagate to New York in about 7 h; additional freshwater discharge along this way is small (Abood, 1977).

In general, high river flow corresponded to low $S$ as expected (Table 2). Depth-averaged $S$ are shown in the context of tidal variability in Figure 5a which shows an inverse correlation between small average $S$ and large tidal salinity fluctuation and vice versa. This is consistent with the length of the salt intrusion varying inversely with river flow and the gradient between oceanic salinities and freshwater varying with the river flow. Note that $v\partial S/\partial y$ is the dominant source term in the salt balance equation.

Figure 5. Observed depth-averaged salinity (a) and depth-averaged streamwise velocity (b) as function of semidiurnal tidal phase, ensemble averages listed in Table 2. Figure 2 provides the key to origin by cruise.
d. Atmospheric forcing

Geyer et al. (2000) discuss current fluctuations in the Hudson River with typical time scale of 2 days and typical magnitude of 0.1 m s\(^{-1}\). These current signals are correlated with water level fluctuations in New York Bight and probably related to regional atmospheric forcing. An extreme example was associated with the above-mentioned severe rainstorm on year day 294 of 1995 (PB00a). Apparently, the cyclone which produced the rain also caused a 0.3 m rise and fall of the low-passed water level seen in Figure 4e. Corresponding to these water level fluctuations, the depth-average residual flow, \(\overline{v}\), was directed up-river while the low-passed water level was rising, and there was net outflow during falling water levels (compare Fig. 4e and Table 2). This table shows that 2–3 day subtidal velocity variations were usually much stronger than the river discharge. They also sometimes overwhelmed the estuarine circulation, making the tidally averaged flow unidirectional with depth during transition and springs of HUDM2 as shown in Figure 7d–e. The estuarine circulation is usually defined as the velocity difference between outflow in an upper layer and inflow in a lower layer. Because the tidally averaged flow \(\overline{v}\) was sometimes unidirectional over the entire water column, Table 2 simply characterizes deviations from the depth average of \(\overline{v}(z)\) in terms of the rms average of \(\overline{v}(z)\) minus its depth average. This is entry \(v_e\).

Figure 7 is based on \(S(z)\) and \(v(z)\) profiles extrapolated parabolically to zero gradients at surface and bottom except for a logarithmic extrapolation of the velocity at the bottom. The extrapolation of \(v(z)\) covers approximately the upper and lower 2.5 m, that of \(S(z)\) about 1–1.5 m at the top and 1 m at the bottom. The extrapolation makes depth-average of \(v\) more realistic while it has minimal effect on depth averages of \(S\).
Based on the above, a further comment can be added on the effects of changes in the length of the salt intrusion on the observations. The length of the salt intrusion can be compared with typical longitudinal displacement resulting from tidal and subtidal currents. Typical tidal current amplitudes of $1 \text{ m s}^{-1}$ (Fig. 5b) correspond to typical longitudinal tidal displacements of about 14 km. Similarly, atmospherically forced subtidal currents with a typical magnitude of $0.1 \text{ m s}^{-1}$ and a typical period around 2 days (Geyer et al., 2000) correspond to longitudinal displacements of less than 10 km. These displacements are

Figure 7. Tidally averaged salinity, (a)–(c), and velocity (d)–(f) as a function of nominal dimensional height above bottom. Profiles have been extrapolated to surface and bottom. Figure 2 provides the key to cruises.
much smaller than typical lengths of the salt intrusion discussed above. Hence, it is expected that indeed all observations stayed within the “central regime” of Hansen and Rattray (1965) as already suggested above.

For partial lack of measurements, economy of presentation and simplicity, effects of the local surface wind stress are not analyzed. In general, the surface stress in the Hudson is much smaller than the bottom stress and has little effect on the tidal and subtidal dynamics (Geyer et al., 2000). However, occasionally wind-related stirring affects mixing in the halocline when it is shallow. Such events are included in the turbulence data presented further below without being specifically identified.

e. Mid-depth characteristics

Flow characteristics from mid-depth are discussed as functions of tidal phase, neap-spring progression and cruise. The discussion of mid-depth conditions is commensurate with a focus on diapycnal mixing in the water column. During ebb, especially spring ebb, stratification tends to be uniform in the vertical (Geyer and Smith, 1987; P97) such that the mid-depth flow and mixing are representative of the water column in general. Under these conditions, the vertical turbulent salt flux at mid-depth is a good measure of the exchange between salty lower and fresh upper layer. Conditions are more complex during flood, however, when a velocity maximum propagates upward through the water column over time (Geyer and Smith, 1987; P97; PB00a). Mixing tends to be weak in the core of the flood jet, where shear is low, as well as in the halocline, where \( N^2 \) is large. Therefore, the turbulent salt flux at the depth of maximum stratification is also discussed. This provides another indicator of mixing across the halocline. The terms cruise-to-cruise variability and longer-term variability are used synonymously.

f. Buoyancy frequency, shear and Richardson number

It is well known that the local stratification and shear affect the generation of turbulence (analyzed in the context of estuarine flow e.g. by Geyer and Smith, 1987 and P97). Analyzing the characteristics of mixing thus requires discussing the variability of stratification, shear and Richardson numbers first. With this narrow focus, it is beyond the scope of this paper to address the competing and interacting effects of freshwater input, tidal stirring and density-driven currents on the stratification. For a conceptual framework, the reader is referred to Simpson et al. (1990), who especially discuss the process of strain-induced periodic stratification (SIPS), to Jay and Musiak (1993), and to MacCready (1999) who models all the above factors. In the absence of strong mixing, the SIPS mechanism results in maximum \( N^2 \) during ebb and minimum \( N^2 \) during flood (Simpson et al., 1990; Jay and Musiak, 1993). Neap observations from the Hudson are consistent with this pattern allowing for some phase delay relative to the depth-averaged tidal flow, while spring tides followed a different pattern. One process making the difference is the weakening of the stratification by intense vertical mixing during spring ebbs (P97).
During neap tides and transitions, mid-depth ensemble average $N^2$ had minima during late flood or slack after flood and maxima during early flood while spring tides had no uniform variability pattern (Fig. 8a–c). While spring floods had minimum $N^2$ during HUDM, they had maximum $N^2$ during HUDM2, and the other cruises show less distinct patterns. Spring ebbs of most cruises were characterized by weak relative minima of $N^2$, the strong ebb-tide minimum of HUDM2 being anomalous, likely related to remote atmospheric forcing and river discharge associated with the year day 294 storm (see above, Fig. 4e and PB00a).

Neglecting detail, $N^2$ generally decreased by factors of 3–10 over the progression from neaps to springs (Fig. 8a–c). With an overall range of $10^{-4}$ s$^{-2}$ $\leq N^2 \leq 2 \times 10^{-2}$ s$^{-2}$, or, in terms of $N$, 6 cph (cycles per hour) to 80 cph, the mid-depth stratification was always strong by oceanic standards. The largest cruise-to-cruise variability occurred during flood tides, longer-term changes in $N^2$ during flood showing a weak correlation with the river

Figure 8. Mid-depth squared buoyancy frequency, squared shear and gradient Richardson number versus semidiurnal tidal phase, ensemble averages listed in Table 2. The key to origin by cruise is given in Figure 2.
The lowest discharge during HUDM corresponds to the lowest $N^2$. Part of the described variations of $N^2$ is also reflected in the average salinity profiles shown in Figure 7a–c. The ensemble-averaged squared shear, $V_z^2$, had magnitudes similar to those of $N^2$, and there was some similarity of variability patterns (Fig. 8d–f). Shear was lower during flood than during ebb, an asymmetry expected from the superposition of tidal and residual flow, which are opposed to each other during flood and in the same direction during ebb. Unlike in $N^2$, this asymmetry held through the fortnightly cycle. The longer-term variability of shear tended to be smaller than that of stratification, especially during spring floods. Cruise to cruise variations of $V_z^2$ were small during ebb tides, especially during spring ebbs.

In order to retain a realistic shear variance in ensemble averages, the mean squared shear
is defined herein as

$$V_z^2 = \left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2,$$

where the $u$ and $v$ that enter the ensemble averaging are 6-min means of the spanwise and streamwise velocity components, respectively. The extra 6-min averaging of the raw 13-s/8-ping ADCP velocity data serves to suppress instrumental noise. As explained above, ensemble averaging is done in 30/45° bins of tidal phase and bins of 0.15 in $z/H$. Note that (2) is consistent with the turbulent kinetic energy equation. An alternative definition of mean shear,

$$\nabla V_z^2 = \left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2,$$

severely underestimates the shear variance available for shear production of turbulence and thus is not used herein (c.f. PB00a). Ensemble average gradient Richardson numbers are computed from the mean squared shear defined in (2),

$$Ri = N^2 V_z^{-2}.$$

The so-defined ensemble average $Ri$ probably is the best nondimensional indicator of the ensemble-average flow state, but owing to the averaging, it is only loosely related to flow instability and mixing. In this average $Ri$, there especially is no equivalent to Miles’ (1961) critical Richardson number of $1/4$. In the following, $Ri \leq 0.25$ are called “small” while $Ri > 1$ is considered “large.”

Over tidal cycles, $Ri$ showed minima during flood and ebb, the first related to low $N^2$, and the second to large shear (Fig. 8g–i). Spring tide $Ri$ were lower than neap tide $Ri$ by almost a decade, a feature attributable to turbulent mixing (P97, PB00b). Longer term variations of flood-tide $Ri$ appear to contain some response to variations in river flow, Figure 9a showing a weak correlation. Minimum $Ri$ during spring ebbs consistently dipped below 0.25 with relatively little variation in magnitude even though the tidal phase of minimum $Ri$ varied (Fig. 8g).

g. Dissipation rate

The viscous dissipation rate is of course the rate of conversion of kinetic energy to heat but it is also a very close measure of the energy flux through the turbulence cascade (Tennekes and Lumley, 1972, p. 20). Hence $\varepsilon$ is the most important and basic characteristic of the turbulence. Given this importance, it appears prudent to establish the reliability and accuracy of the observed $\varepsilon$. Comments are especially in place because $\varepsilon$ was often very large such that the microstructure shear only resolved the inertial subrange and not the viscous range of the turbulence spectrum (P97).

Firstly, the observed $\varepsilon$ was consistent with the overall energetics of the tidal flow as
shown by PB00a. The depth-integrated viscous dissipation rate was closely related to the work done by the streamwise pressure gradient adjusted for the rate of change of turbulent kinetic energy such that the local energy budget was approximately closed. Herein, the longitudinal pressure gradient was measured using moored pressure gauges by Geyer et al. (2000).

Secondly, PB00a also demonstrate that, as expected, \( \varepsilon \) stayed close to the law-of-the-wall dissipation rate in the approximately 1 m thick logarithmic layer (Trowbridge et al., 1999) above the bottom. Most microstructure profiles reached to within 0.15 m above the bottom such that \( \varepsilon \) can be determined near the bottom following Dewey et al. (1987). The law-of-the-wall dissipation rate is found from the bottom friction velocity \( u^* \)

\[
\varepsilon_b = \frac{u^3}{(kz)},
\]

where \( k = 0.4 \) is von Kármán’s constant, and \( u^* = \sqrt{\tau_b/\rho} \) with bottom shear stress \( \tau_b \) and density \( \rho \). The bottom shear stress is estimated from ADCP velocities \( u_b \) about 2.25 m above the bottom as \( \tau_b = \rho c_d |u_b| u_b \) with drag coefficient \( c_d = 0.002 \) based on Trowbridge et al. (1999). There were systematic deviations from the vertical structure of neutral boundary layers resulting in some uncertainty of the value of \( c_d \), \( c_d = 0.0025 \) still being consistent with the Trowbridge et al. (1999) measurements. Data from HUDM and HUDM2 show approximately lognormal distributions of \( \varepsilon/\varepsilon_b \) at \( z = 0.275 \) m with median values of 0.8–0.9 and arithmetic means of 1.35–1.42. This excludes data from slack tides.

With respect to the depth-time structure of \( \varepsilon \), PB00a note that the similarity and correlation of \( \varepsilon \) and \( \varepsilon_b \) decreased with distance from the bottom. Most noteworthy, the actual \( \varepsilon \) regularly exceeded \( \varepsilon_b \) during spring ebb when \( Ri \) was small. During neap ebbs with large \( Ri \), \( \varepsilon \) tended to be much smaller than \( \varepsilon_b \). The following discussion further illustrates that the local mixing well above the bottom was more than a simple function of the bottom shear stress.

Over tidal cycles, \( \varepsilon \) varied with a range of about 2 decades. In a bit of simplification, semidiurnal variations with flood-tide maxima and ebb-tide minima during neaps gave way to quarter diurnal variations with a weaker flood maximum and a dominant ebb maximum during spring tides (Fig. 10). As shown previously by Geyer and Smith (1987) and P97, the low \( \varepsilon \) during neap ebbs is related to suppression of turbulence by stable stratification given large \( Ri \), while the high \( \varepsilon \) of spring ebbs apparently results from vigorous shear instability at small \( Ri \) (compare Fig. 8g–i). Over the progression from neaps to springs, \( \varepsilon \) increased prominently by 2 to almost 3 decades during ebb, while flood values did not change systematically. For further quantitative characterization of tidal changes, harmonic fits of log10(\( \varepsilon \)) at semidiurnal and quarter diurnal frequencies are listed in Table 3. Note especially the changes from dominantly semidiurnal to dominantly quarter diurnal variation.

Cruise-to-cruise variations of \( \varepsilon \) were large except during spring ebbs, when \( \varepsilon \) ranged within only a factor of 3 and when shear and \( Ri \) also varied comparatively little. Ebb maxima occurred over a considerable range of tidal phases, however. Outside of spring
ebbs, longer-term variations of $\varepsilon$ reached up to factors of 20, neap tides showing more prominent variability than spring tides.

Figure 10 depicts confidence limits from all data plotted together, which allow a rough assessment of how large variations in $\varepsilon$ have to be in order to be statistically significant.

Figure 10. Mid-depth viscous dissipation rates versus semidiurnal tidal phase. The key to origin by cruise is given in Figure 2. Upper and lower confidence bounds from all data are plotted in the lower right hand corner.
Table 3. Harmonic fit of log_{10} (ε), log_{10} (K_m) and log_{10} (K_p), mean (M), semidiurnal (SD) and quarter diurnal (QD) amplitudes.

<table>
<thead>
<tr>
<th>Cruise</th>
<th>Category</th>
<th>log_{10} (ε)</th>
<th>log_{10} (K_m)</th>
<th>log_{10} (K_p)</th>
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</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>M  SD  QD</td>
<td>M  SD  QD</td>
<td>M  SD  QD</td>
</tr>
<tr>
<td>HUD1</td>
<td>transition</td>
<td>-5.2 0.3 0.3</td>
<td>-2.9 0.5 0.4</td>
<td>-3.3 0.5 0.5</td>
</tr>
<tr>
<td>HUD1</td>
<td>springs</td>
<td>-5.4 0.4 0.5</td>
<td>-2.7 0.3 0.2</td>
<td>-3.2 0.1 0.4</td>
</tr>
<tr>
<td>HUD2</td>
<td>neaps</td>
<td>-5.7 0.7 0.3</td>
<td>-3.2 0.6 0.3</td>
<td>-3.8 0.9 0.4</td>
</tr>
<tr>
<td>HUD2</td>
<td>springs</td>
<td>-5.3 0.2 0.8</td>
<td>-2.4 0.5 0.3</td>
<td>-2.7 0.3 0.7</td>
</tr>
<tr>
<td>HUD3</td>
<td>neaps</td>
<td>-5.7 0.3 0.4</td>
<td>-3.1 0.5 0.7</td>
<td>-3.8 0.2 0.6</td>
</tr>
<tr>
<td>HUD3</td>
<td>transition</td>
<td>-5.5 0.2 0.5</td>
<td>-3.0 0.5 0.6</td>
<td>-3.5 0.3 0.6</td>
</tr>
<tr>
<td>HUD3</td>
<td>springs</td>
<td>-5.3 0.2 0.8</td>
<td>-2.6 0.2 0.5</td>
<td>-2.9 0.2 0.8</td>
</tr>
<tr>
<td>HUDM</td>
<td>neaps</td>
<td>-5.3 0.5 0.2</td>
<td>-2.6 0.7 0.3</td>
<td>-3.1 0.9 0.3</td>
</tr>
<tr>
<td>HUDM</td>
<td>springs</td>
<td>-5.1 0.2 0.7</td>
<td>-1.9 0.6 0.6</td>
<td>-2.3 0.2 0.7</td>
</tr>
<tr>
<td>HUDM2</td>
<td>neaps</td>
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<td>-3.0 0.6 0.3</td>
<td>-3.7 0.9 0.0</td>
</tr>
<tr>
<td>HUDM2</td>
<td>transition</td>
<td>-5.4 0.5 0.6</td>
<td>-2.4 0.2 0.2</td>
<td>-3.1 0.4 0.7</td>
</tr>
<tr>
<td>HUDM2</td>
<td>springs</td>
<td>-5.2 0.3 0.5</td>
<td>-2.5 0.2 0.0</td>
<td>-2.9 0.4 0.4</td>
</tr>
</tbody>
</table>

Note that most uncertainty limits are smaller than a factor of 2 up or down, and that longer-term changes in ε were often much larger than a factor of 2. Upper and lower confidence limits are plotted as small minus signs relative to a central “plus” at 5 × 10^{-7} W kg^{-1}. The confidence limits were computed by bootstrapping (Efron and Gong, 1983) at the 97% confidence level.

h. *Estimating the turbulent momentum and salt flux*

From the measured ε, shear and stratification, eddy viscosity $K_m$ and eddy diffusivity $K_p$ as well as vertical turbulent fluxes of momentum, $J_m$, and salt, $J_s$, can be estimated as follows. Here, a brief summary is presented while PB00b contains more detailed reasoning.

$$K_m = (1 - R_f)^{-1} \varepsilon \nu \zeta^{-2} \, [m^2 \cdot s^{-1}], \quad (6)$$

$$K_p = R_f (1 - R_f)^{-1} \varepsilon \zeta^{-2} \, [m^2 \cdot s^{-1}], \quad (7)$$

$$J_m = -\rho K_m \partial u / \partial z \, [Pa], \quad (8)$$

$$J_s = -10^{-3} \rho K_p \partial S / \partial z \, [kg \cdot m^{-2} \cdot s^{-1}]. \quad (9)$$

The factor $10^{-3}$ in (9) converts practical salinity $S$ to concentration units. The approach is based on Busch (1977) and Osborn (1980).

The method of Osborn (1980) is well established in the oceanic microstructure literature where it has been used under the assumption that $R_f$ is constant and attains a maximum value. The latter is usually quoted in the form of the (misnamed) “mixing efficiency” $\Gamma = R_f / (1 - R_f)$. With Osborne’s claim of $\Gamma \leq 0.2$, setting $\Gamma = 0.2$ has been most common. The first attempt at determining $\Gamma$ was carried out by Oakey (1982) who found $\Gamma = 0.24 \pm$
0.14. Later investigations include direct heat flux measurements by Moum (1996) who states a range of $\Gamma$ of 0.04 to 0.4. Laboratory measurements also do not constrain $\Gamma$ well. The shear flow experiments of Rohr et al. (1988), e.g., show $\Gamma = 0.19$ to 0.31 at $Ri$ between 0.18 and 0.2 (Fig. 11).

The uncertainty of the level of $\Gamma$ is obviously important in attempts at estimating the turbulent salt flux. The approach here is to follow the tradition of oceanic microstructure work as much as possible and to subsequently compare the estimated $J_S$ with vertical salt fluxes derived from salt conservation in the estuarine flow.

The tidal flow of the Hudson River shows extended periods of small gradient Richardson numbers $Ri < 0.1$ both in the weakly stratified bottom layer and in the strongly stratified and strongly sheared water column during spring ebbs (P97). Because of these small $Ri$, it is impermissible here to assume constant $R_f$. Note that the buoyancy term of the turbulent kinetic energy (TKE) equation vanishes at $Ri \to 0$ but not the shear production, and thus $R_f$ has to vanish as $Ri \to 0$. Consequently, the flux Richardson number,

$$R_f = K_p N^{-2} (K_m V_m^2)^{-1},$$

is a prescribed function of the gradient Richardson number, $R_f = R_f (Ri)$ (Fig. 11). The limits of this function are $R_f \to 0$ for $Ri \to 0$ and $R_f \to R_f^\infty = 0.19$ for $Ri \to \infty$. The latter

Figure 11. Modified Osborn (1980) method: coefficients of eddy viscosity, $1/(1 - R_f)$, and of eddy diffusivity, $R_f/(1 - R_f)$, as function of Richardson number. Laboratory observations by Rohr et al. (1988) are given for comparison (plus signs).
makes levels of $R_f$ (or $\Gamma$) comparable to past usage for $Ri \geq 0.2$. Technically, instead of $R_f$, the turbulent Prandtl number $Pr_t = K_m/K_r$ is specified in modification from Schumann and Gerz (1995), as

$$Pr_t(Ri) = \frac{K_m}{K_r} = Pr_t^0 \exp \left( \frac{-Ri}{Pr_t^0 R_f^\infty} \right) + \frac{Ri}{R_f^\infty}$$  \hspace{1cm} (11)

with $Pr_t^0 = 0.63$. The flux Richardson number then follows from (10).

The eddy viscosity and the turbulent momentum flux are insensitive to the modeling of $R_f(Ri)$ owing to the limited variability of $R_f$ in (7) and as shown in Figure 11. At $Ri \leq 0.1$, the chosen $R_f(Ri)$ fits the laboratory observations of Rohr et al. (1988) well. Maintaining a finite $R_f$ at asymptotically large $Ri$ is possibly unrealistic. This is inconsequential, however, because little mixing is associated with large $Ri$. Setting $R_f = 0$ for $Ri > 1$ in a sensitivity test reduces ensemble average $J_S$ by typically less than 10%. The estimated salt flux is most sensitive to values of $R_f$ in the range of $Ri$ which carries the bulk of mixing, $0.05 \leq Ri \leq 0.5$. During HUDM2 at mid-depth, e.g., half of the variance contributing to the mean salt flux came from $Ri < 0.25\%$ and 75% from $Ri < 0.5$. The problem of modeling $R_f$ thus rests with the value of $R_f^\infty$.

The method of Osborn (1980) is based on a steady state production-dissipation balance in the TKE equation. The validity of this assumption is supported by numerical modeling of estuarine mixing using a $k-\varepsilon$ closure currently undertaken by myself and H. Baumert, a topic discussed in more detail in PB00b. Eddy coefficients and fluxes are computed from individual microstructure drops, using 6-min ADCP velocity averages and $N^2$ computed from Thorpe-sorted (1977) potential density such that $N^2 \geq 0$. Averaging these drop-based $K_m$, $J_m$ etc. yields the ensemble averages analyzed herein (c.f. PB00b).

### i. Eddy viscosity and eddy diffusivity

Eddy coefficients show temporal variations similar to $\varepsilon$. Table 3 quantifies semidiurnal and quarterdiurnal variations of $K_m$ and $K_r$. As in the case of $\varepsilon$, semidiurnal variability tended to dominate during neaps, changing to dominant quarter diurnal variability during springs. Avoiding redundant discussions, this section focuses on the depth and longer-term variability of $K_m$ and $K_r$. Tidally averaged $K_r$ and $K_m$ tended to decrease exponentially with height from maxima near $z = 5$ m, the decrease being 1 to over 2 decades and tending to be stronger in $K_r$ than in $K_m$ (Fig. 12c,d). The difference between the two eddy coefficients is related to the prescribed $Pr_t(Ri)$ combined with a general increase of $Ri$ toward the surface. ADCP measurements do not cover the lowest 2.5 m above the bottom, and thus there are no observations of any expected decrease in $K_r$ and $K_m$ toward the bottom. While Figure 12c,d depicts averages over tidal cycles, Figure 12a,b shows ensemble averages in 30°–45° bins of $\Phi_{sd}$ such that tidal variations are retained.

Figures 12c,d contain both the fortnightly and the longer-term variability of tidally averaged $K_r$. The overall variability range increased with height $z$. Among the five cruises, HUDM had the largest $K_r$ and $K_m$, while HUD3 the smallest. These cruise-to-cruise
variations ranged from factors of 2 to almost 20. Both cruises took place during small river flow, small shear and relatively weak stratification. Hence, there is no simple explanation for the longer-term variations of $K_m$ and $K_p$. Note further that fortnightly changes in tidally averaged $K_m$ and $K_p$ were significant as one would expect from the underlying variations in $\varepsilon$ displayed in Figure 10.

Tidally averaged eddy coefficients were computed from ensemble averages in $\Phi_{SD}$ bins as summarized in Table 2. The statistical uncertainty limits of $K_m$ and $K_p$ plotted in Figure 12c,d are rather wide in response to large tidal signals. The limits reflect the question of how sensitive the averages are to omissions of parts of tidal cycles. As most tidal cycles were covered well, the uncertainty limits are conservative.

Figure 12. Eddy diffusivity and eddy viscosity as function of actual height above bottom, ensemble averages as defined in Table 2 (a–b) and with additional averaging over tidal cycles in c–d. Statistical uncertainty limits of all tidally averaged data are shown in the lower left corner of c and d. Figure 2 depicts the key to origin by cruise.
j. Momentum—stress

The turbulent momentum flux and the stress are related through Newton’s second law as \( \tau = -\mathbf{J}_m \). Following common usage, a discussion of \( \tau \) instead of \( \mathbf{J}_m \) follows. The focus is on the streamwise stress component \( \tau_y \), Figure 13 depicting the mid-depth \( \tau_y \). The semidiurnal variability of turbulent stress with reversal of sign between ebb and flood is obvious, but it is the neap-to-spring increase in stress magnitude that catches the eye. Ebb-tide \( |\tau_y| \) ranged from 0.01 to 0.08 Pa during neaps, while spring tide values reached up to 0.75 Pa. This increase in ebb-tide \( |\tau_y| \) was paralleled by a decrease in \( Ri \) to subcritical values depicted in Figure 8g–i. There was no uniform fortnightly increase in flood-tide \( \tau_y \) even though the largest stress did occur during spring flood. Note that large \( \tau_y \) of up to 0.5 Pa occurred even during neap floods.

Cruise-to-cruise variations of stress during flood tides had a range of partially over one decade (Fig. 13), and the ebb-tide turbulent stress also varied substantially during neaps and transitions. In contrast, maxima of \( |\tau_y| \) during spring ebbs were remarkably uniform in cruises HUD2-HUDM2. Only HUD1 showed much smaller stress magnitudes than the other cruises.

Given a general dependence of water column turbulence on the bottom shear stress as discussed in PB00b, the mid-depth turbulent stress should be affected by the magnitude of \( \tau_b \). Normalizing the mid-depth stress by the bottom stress may then reveal other factors influencing the mixing. Indeed, the normalized stress \( \tau_y/\tau_{by} \) was inversely correlated with \( Ri \) during floods, especially during spring floods (Fig. 14). This correlation clearly demonstrates a damping effect of stable stratification on the turbulence. In contrast, there was much less correlation of normalized stress with \( Ri \) during ebbs, and none during spring ebbs. Note that spring ebbs had only small variation in mid-depth \( Ri \). During HUD1 springs, both \( |\tau_y| \) and \( \tau_y/\tau_{by} \) were unusually small, a finding without simple explanation. HUD1 spring conditions had the smallest \( S \) of all cruises (Fig. 7a), but mid-depth \( N^2 \) and shear did not differ strongly from the other cruises (Fig. 8).

Geyer et al. (2000) have analyzed the integral momentum budget in the same stretch of the Hudson River discussed in this paper. Their work is based on a central ADCP/CTD mooring and two pressure gauges about 3 km upstream and downstream, respectively. Time series in August–October 1995 had 2½ months duration. Figure 15 shows their estimates of 2.5-month average maximum ebb-tide and flood-tide streamwise “stress” during spring and neap tides. This stress really is the residual of the momentum balance; small-scale turbulence is only part of it, and other processes such as lateral circulations may contribute also. Superimposed in Figure 15 are estimates of the turbulent stress from HUDM and HUDM2, which fall toward the beginning and the end of the 2.5-month mooring period, respectively. Each curve represents an average of two semidiurnal cycles from two consecutive days (Table 2). The stress estimates from the momentum integral and from the turbulence measurements thus had radically different sampling.

Note that, in accordance with the above and in the absence of sampling mismatches, stress estimates for momentum integral provide upper bounds for the estimates from the
Figure 13. Mid-depth turbulent stress versus tidal phase. The key to origin by cruise is given in Figure 2.

turbulence measurements. Figure 15 shows that the two different stress estimates coincide very closely during spring ebbs and also match quite well during HUDM neap flood. Neap ebbs as well as HUDM2 spring flood show turbulent stress of about $\frac{1}{3}$ of the estimate from the momentum integral, and the turbulent estimate exceeds the integral estimate during
HUDM2 neap flood. This latter anomaly appears to be related to the substantial inflow into the estuary at the time (Table 2; Fig. 7f) resulting in rather large bottom stress and stress in the water column (PB00b). In summary, the momentum integral of Geyer et al. (2000) and the turbulent stress estimates of this paper are highly compatible with each other.

k. Mid-depth salt flux

The variability of the mid-depth vertical turbulent salt flux (Fig. 16) was similar to that of $\varepsilon$ (Fig. 10), tidal variations of $J_S$ changing from dominantly semidiurnal during neaps to dominantly quarter diurnal during springs. The overall largest $J_S$ of $5 - 10 \times 10^{-4}$ kg m$^{-2}$ s$^{-1}$
occurred during spring ebbs. There was a strong increase in $J_S$ from neaps to springs during ebb but not during flood. Flood tide maxima of $J_S$ showed large cruise-to-cruise variations with a range of $1 - 6 \times 10^{-4}$ kg m$^{-2}$ s$^{-1}$. Neap ebbs also had significant longer-term variations of $J_S$, HUDM2 exhibiting exceptionally low $J_S$. In contrast, maximum $J_S$ during spring ebbs spanned a range of only a factor of 2, HUD1 displaying the smallest flux.

Similar to turbulent stress, $J_S$ exhibited some response to variations of $Ri$ (not shown). During floods, the correlation of $Ri$ and $J_S$ was lower than that between $Ri$ and $\tau_y$. The increase of ebb-tide $J_S$ from neaps to springs paralleled the decrease in $Ri$, but as with $\tau_y$, there was no correlation of $J_S$ and $Ri$ during spring ebbs. The maximum of $J_S$ at $\Phi_{SD} = 225^\circ$ in Figure 16c occurred during HUD3 in a topographically forced event discussed in Section 5.

Much of the cruise-to-cruise variability of the mid-depth turbulent salt flux disappears when $J_S$ is integrated as function of time. Table 4 shows $\int_{SD}^{260} c_j J_S (\Phi_{SD}) d\Phi_{SD}$ for full tidal integrals and $\int_{90}^{270} c_j J_S (\Phi_{SD}) d\Phi_{SD}$ for ebb-only integrals, respectively. Here, $c_j = 12.4 \times 3600$ s/360° converts phase to time. The cruise-to-cruise variability range of the time-integrated salt flux was only a factor of 2. Spring tidal cycles provided 1.7–3 times more vertical salt flux than neap tidal cycles, and ebb tides carried 70–80% of the total tidal salt flux during springs.

Examining the integral salt balance is more difficult than analyzing the integral

Figure 15. Maximum streamwise stress during ebb and flood and neaps (dashed) and springs (solid) from the integral momentum balance of Geyer et al. (2000) (thick lines), HUDM (thin lines) and HUDM2 (thin lines with x-symbols).
Figure 16. Mid-depth turbulent salt flux versus tidal phase. The key to origin by cruise is given in Figure 2. The peak at $\Phi_{SD} = 225^\circ$ in (c) is related to the first event discussed in Section 5.
momentum balance. Using the same observations as in Geyer et al. (2000), Geyer (personal communication, 1999) estimates the 2.5-month August–October 1995 average mid-depth upward salt flux as $1.5 \times 10^{-4}$ kg m$^{-2}$ s$^{-1}$, or 6.7 kg m$^{-2}$ in 12.4 h. Larger tidally-averaged fluxes of up to $4 \times 10^{-4}$ kg m$^{-2}$ s$^{-1}$ occurred during neap tides. These flux values are the sum of advective transport by vertical velocity and turbulent diffusion. The moored observations did not allow estimating the vertical velocity to an accuracy sufficient to extract the advective part from the total vertical salt flux.

The above average total salt flux of 6.7 kg m$^{-2}$ from salt conservation compares generally well with the turbulent salt flux estimates given in Table 4. Averaging over the fortnightly cycle during HUDM2 yields 7.2 kg m$^{-2}$, while HUDM had a larger salt flux. Further comparisons are difficult because of the mismatch in sampling between microstructure data and integral budget. Considering that vertical advection must have carried some fraction of the total vertical salt flux, it appears possible that the microstructure-based salt flux estimates might be somewhat too large. Note that if the specification of $R_f = R_f(R_i)$ had been based on Rohr et al. (1988) (Fig. 11), values of $J_S$ would have been larger than as discussed above.

1. Salt flux across the halocline

The depth of the halocline varied substantially over tidal and fortnightly periods (P97, PB00a), and thus the turbulent salt flux is estimated at the halocline depth in order to gain an alternative measure of the exchange between the salty lows and fresh upper layers. The halocline depth is defined as the depth of maximum $N^2$. Normalized heights above bottom $z/H$ of maximum $N^2$ had a distribution with peak at 0.79, mean of 0.77, and few values below 0.6 or above 0.86. With a nominal average water depth of 16 m, the halocline was at a depth of only 3.5 m on average.

P97 describes mixing across the halocline as “weak” and characterizes the turbulent salt flux as “insignificant.” These characterizations fail to take into account that even weak mixing can add up over time and that spring ebbs represent only a small portion of the fortnightly cycle. Figure 17 depicts the time-integral of the salt flux in the halocline for all days of observations. The salt flux in the halocline had the same order of magnitude as the mid-depth flux (compare Figure 17 with Table 4). Contributions from spring ebbs are very
prominent, but the graph also shows that significant contributions occurred during flood and ebb at all phases of the fortnightly cycle.

The time-integral $\int j_s dt$ shows some step-like increases. Figure 17 labels as “wave” an event related to topographic forcing analyzed in Section 5 and shown in Figure 21. Other events occurred when winds were strong and the halocline was very shallow (not shown). In these cases, the surface wind stress accelerated the shallow fresh layer and caused high shear and flow instability in the halocline.

4. A long streamwise section

Focusing on the temporal variability of mixing in the relatively uniform section of the river at $3 \leq y \leq 8$ km, we tried only once to observe variations along the river. On August 26, 1995, during ebb, we took a long section from above the George Washington Bridge to just north of a trench across the river which is featured in Section 5 below (Fig. 18b). This section shows interesting variability of flow and mixing and allows some conclusions on the spatial distribution of mixing.

The drift began near the deepest part of the river, the thalweg, at $y = 12.6$ km, but currents and wind pushed the ship westward toward the New Jersey shore and into gradually shallower water. Near $y = 8$ km, the ship was in such shallow water that we repositioned toward deeper water, a procedure repeated at $y = 5.2$ km and $y = 4.5$ km. Toward the southern end of the section, we encountered problems with the CTD component of SWAMP and terminated the section. During its 2 h 48 min duration, the ebb began to wane. The temporal decrease of ebb currents was slightly amplified by the seaward drift of the R/V Onrust (Fig. 18a), the latter being concluded from a comparison of locally observed depth-averaged streamwise currents with the water level at the Battery. The observations thus mix streamwise, spanwise and temporal variability.
The northward end of the section shows some obvious flow variability associated with the variable bathymetry near the George Washington Bridge, where depths partially exceed 20 m (Fig. 19). Contours of $v$ bend downward in the deep river section, and the $u$ velocity component attains significant magnitude near $y = 11$ km in relationship to the bending of the channel. South of the deep stretch, shear and stratification increased and were rather uniform with depth, typical for ebb. While currents decreased toward the southern end of the section, shear and stratification did not. The seaward increase in salinity in the lower half of the water column was non-uniform.

Squared shear attained a relative maximum near $y = 8.5$ km, where the ship was west of the thalweg and where the water depth was correspondingly small (Fig. 19). Here, depth-averaged $Ri$ attained a minimum corresponding to the large shear. At $y > 8$ km, high dissipation rates visually appear correlated with low $Ri$ and high shear. Such correlation did not exist at $y < 6$ km, however, where $\varepsilon$ decreased to low values while shear stayed high and $Ri$ stayed low.

Despite problems with mixed spatial and temporal variability, some conclusions can be drawn from the long streamwise section.

1. Dissipation rates were well within the range of other $\varepsilon$ measured farther south of this section during similar flow conditions. Similarly, the turbulent salt flux (not shown) was comparable to the remainder of the Hudson data.

2. Energetic turbulent mixing was distributed over a wide area as shown in Figure 19.

3. The gradient Richardson number is a poor quantitative indicator of mixing.
The first statement is based on a quantitative comparison of $\varepsilon$ and $J_z$ from the long section with other data from strong ebbs. The third statement can be illustrated by displaying two-dimensional averages of the buoyancy Reynolds number $Re_b = \varepsilon/\nu N^2$ as a function of streamwise location $y$ and Richardson number (Fig. 20). Even though the largest $Re_b$
tended to occur at low \( Ri \), \( Re_b \) was clearly also a function of \( y \). Note especially the vertical contours at \( 5 \text{ km} < y < 7 \text{ km} \), an area where low \( Ri \) combines with low \( \varepsilon \). Similar systematic shifts in relationships between \( Ri \) and \( Re_b \) also occurred as a function of time (not shown).

5. Flow across a trench

A pipeline crosses the Hudson River at \( y \approx 3.95 \text{ km} \) (Fig. 1), being buried partly in a trench 2–3 m deep and 100 m wide. Near the Manhattan shore, the trench gives way to a dike (J. Trowbridge, personal communication, 1997). At times, the trench forces striking “hydraulic” responses of the river flow. Cruise HUD3 encompasses two detailed sets of observations, the first taken on ebb when the topographically forced flow response had the form of a train of first vertical mode internal lee waves (Fig. 21). The second observation had less wave activity but a vertical halocline displacement above the trench which greatly exceeded the depth of the trench below the surroundings (Fig. 22). In both cases, enhanced turbulent dissipation occurred above and downstream of the trench. Vertical velocities \( (w) \) reached \( \pm 0.1 \text{ m s}^{-1} \), and were readily observable with the ADCP.

The observations shown in Figure 21 were taken during neap tides on year day 99, 1995, toward the end of ebb (see also Fig. 4c). The ship was drifting seaward with a rather uniform velocity of \( -1.5 \text{ m s}^{-1} \), while depth-averaged streamwise currents ranged from \( -0.68 \text{ to } -0.79 \text{ m s}^{-1} \). Wind speeds, measured at Newark Airport 20–25 km southwest of the study area, were modest at 4–5 m s\(^{-1}\), and thus ship drift and surface current coincide well. The halocline dipped downward over the trench, and the pattern of vertical velocity changed abruptly 50 m ahead (northward) of the trench into internal waves obviously of

Figure 20. Average buoyancy Reynolds number as a function of Richardson number and streamwise location in the section depicted in Figure 18.

[Image of Figure 20]
Figure 21. Streamwise velocity (a), vertical velocity (b), salinity (c), squared shear (d), gradient Richardson number (e), and viscous dissipation rate (f) near a trench across the Hudson River during ebb at 1449 UT–1500 UT, April 9, 1995, cruise HUD3. Note the train of first mode internal waves with vertical velocities of up to ±0.1 m s\(^{-1}\) and the enhanced \(\epsilon\) over the downstream side of the trench and farther away in the halocline. Shading indicates the river bed; stars mark microstructure drops. Pressure in dbar and depth in m coincide numerically to better than 1%.
Figure 22. Flow over a trench across the Hudson River as in Figure 21 but during flood at 2139 UT–2155 UT on April 11, 1995, cruise HUD3. Large “X” indicate visually observed surface rip.

vertical mode 1 with 100 m wavelength (Fig. 21b). This horizontal wavelength is consistent with the streamwise scale of the trench. Wave amplitudes gradually decreased downstream (southward).

Stratification was strong and concentrated in a well-defined halocline. Figure 21c
suggests that the halocline deepened and sharpened upon the flow response. Maximum $N^2$ of $2.5 - 5 \times 10^{-2}$ s$^{-2}$ occurred at 2–5 m depth off the trench and as deep as 7 m above the trench. Both $N^2$ and shear were smaller near the trench than ahead and behind it. Shear increased behind the trench and was modulated by the wave packet (Fig. 21d).

One wavelength in the wave packet was observed in about 1 min, and thus little time-averaging of the ADCP was possible. In order to suppress the substantial instrumental noise in vertical shear, velocity profiles were fit with smoothing cubic splines, and $N^2$ was treated similarly to match. Hence, vertical resolution in shear, $N^2$ and $Ri$ is limited. Moreover, the microstructure drops were spaced about 100 m apart and thus do not resolve the wave structure even though we profiled at near-maximum rates of about 1/min. Possible modulations of $Ri$ and $\varepsilon$ by the wave train thus remain unexplored. The structure of $Ri(y, z)$ as depicted in Figure 21e shows low values near the bottom, high values near the surface, and intermittent values in the halocline. Beyond this coarse description, $Ri(y, z)$ does not resolve the actual flow state.

Figure 21f shows enhanced $\varepsilon$ above the trench as well as in the halocline some 400 m south of it. The maximum in the halocline is more than a curiosity because it occurred in a region of very large salinity gradient such that the turbulent salt flux became as large as $10^{-2}$ kg m$^{-2}$ s$^{-1}$, large enough to clearly show up at $\Phi_{M2} = 225^\circ$ in the ensemble average $J_S$ shown in Figure 16c.

The second set of measurements at the trench deviates from the first in a lack of obvious wave activity (Fig. 22). This section was taken during neap tides and decelerating flood on year day 101, 1995 (compare Fig. 4c). Internal waves were not completely absent as shown by $w(y, z)$, but they were not organized in a single coherent wave train but rather spread over a range of wavelengths. Most prominent is a wave-like feature with troughs at the trench and at $y = 4.5$ km. The flow convergences near the wave troughs were visible on the surface by a modulation of small-scale surface gravity waves. The “X” in Figure 22a mark maximum surface roughness. The streamwise wavelength of this feature is 500 m, much larger than the width of the trench.

Figure 22a shows the flood jet with about 1.2 m s$^{-1}$ speed, located close to the surface as expected for late flood. Surface velocities were about 1 m s$^{-1}$ with slower speeds above the trench. More so than in the first event, the river flow was three-dimensional with prominent westward velocity above the trench (not shown). The depth-averaged flow was about 0.9 m s$^{-1}$ and thus slightly larger in magnitude than in the first event shown in Figure 21. Compared with the first event, the second event had slightly smaller maximum $N^2$ in the halocline, and $N^2$ was more variable below. Dissipation rates depicted in Figure 21f and 22f both show maxima above and slightly downstream of the trench. The second event did not reach the very large $J_S$ in the halocline of the first event, however.

Thus, how important was the local enhancement of mixing by topographic forcing within the regional estuarine circulation? In the second event, Figure 22, the overall average of $\varepsilon$ (of all drops displayed) is 60% larger than when the 2 drops with the highest $\varepsilon$ are omitted. Within the displayed drop sequence near the trough, topographically enhanced
mixing was important. However, the enhancement was not large enough to significantly affect spatial averages over the larger surroundings, say \(2 \text{ km} < y < 8 \text{ km}\).

Mixing was more strongly enhanced in the first event, Figure 21; its effect being seen in the ensemble averages displayed in Figures 16c and 10c (c.f. Section 3). However, these averages have to be taken with a grain of salt because they result from irregular space-time sampling. The trench at \(y = 3.9 \text{ km}\) was not sampled during HUD1, HUDM and HUDM2 while coverage was irregular during HUD2 and HUD3. In assessing the impact of enhanced mixing near the trench, one has to consider the limited area affected as well as the question of how often flow conditions are such that \(\varepsilon\) is substantially enhanced as in Figure 21. These conditions require critical flow, i.e. phase speed equaling effective mean flow speed, combined with resonant forcing. Resonance means that the vertical structure implied by the dispersion relationship at the given frequency \((\omega = 0)\) and horizontal wavelength (100 m) happens to match the water depth. One may suspect that such conditions appear only during short parts of the tidal cycle, but this has to be confirmed by measurements. Linear two-layer theory as well as neutral linear vertically-continuous wave solutions fail to predict realistic dispersion characteristics of internal lee waves.

6. Summary and discussion

Five microstructure cruises in the Hudson River took place under widely varying conditions of river discharge and external forcing by water level fluctuations in New York Bight. Nevertheless, tidal and fortnightly variability patterns of stratification, shear and mixing characteristics were qualitatively repeatable, largely following the description of Peters (1997).

— Longer-term variations of \(N^2\) and \(R_i\) during flood were weakly correlated with the river discharge.

— **All flood tides had substantial mixing.** Cruise-to-cruise variations of mixing parameters \(\varepsilon, J_S\) and \(\tau_y\) were large and correlated with variations in \(R_i\).

— **During ebb tides, mixing was weak in response to stable stratification during neaps, and it increased dramatically toward spring tides** in parallel with the occurrence of low \(R_i\). Cruise-to-cruise variations of mixing became small toward spring tide.

— The largest vertical salt flux occurred during spring ebbs, which provided about 30% of the total fortnightly salt flux, floods throughout the fortnightly cycle providing most of the remainder.

— The observed “level” of small-scale mixing, i.e. stress and vertical salt flux, was consistent with the integral momentum and salt budgets in the uniform segment of the Hudson where the bulk of the measurements were taken.

— One topographically forced flow feature examined, lee waves behind a trench across the river, was associated with some elevated mixing.
This paper, in conjunction with others based on Hudson cruises (P97, PB00a, PB00b), represents a first glance at the mixing climate of one estuary. There are few directly comparable observations from estuaries, results from recent European microstructure measurements not having been published yet. Most akin to this study are the observations of Reynolds stress and turbulent shear production with an ADCP of Stacey et al. (1999). Their measurements were taken over one 25 h period in San Francisco Bay with tidal velocities smaller than reported here. Consequently, production rates, approximately equal to dissipation rates, were also smaller than in the Hudson. The space-time evolution of turbulence was very similar in both estuaries, however. Lu et al. (2000) also used an ADCP to measure turbulence in a tidal channel in British Columbia, and they additionally made moored microstructure observations at mid-depth. Even though the channel is affected by river run-off, the rather complex, three-dimensional flow bore only limited resemblance to more two-dimensional estuarine circulation patterns. Dissipation rates again were somewhat smaller than reported here in response to smaller tidal velocities. Eddy viscosity and eddy diffusivity tended to be larger than in the Hudson, however, owing to much smaller vertical velocity and density gradients (c.f. (7) and (8)).

In recent years, turbulence measurements have become common on the continental shelf, and results bear resemblance to the findings herein to a degree owing to the common influence of tides. Simpson et al. (1996) examine mixing at both mixed and stratified sites in the Irish Sea. Even in the latter case the variability of turbulent mixing was amazingly regular and easily and successfully simulated in turbulence closure modeling. Dissipation rates were somewhat smaller than reported here, but share one important feature documented in P97 but not herein. Turbulent mixing, as characterized by $\varepsilon$, showed a phase lag relative to the tidal current, the lag increasing with distance from the bottom. This finding held even during strongly stratified conditions of ebb. It demonstrates the overall importance of the bottom shear stress for estuarine mixing. The finding does not imply that turbulence was dominantly exported from the bottom boundary layer (PB00a).

Other observations of coastal mixing show complex, rather than simple space-time variability patterns of turbulent mixing. Examples are studies of mixing on Georges Bank in the vicinity of a tidal front in water depths of about 60 m (Horne et al., 1996; Yoshida and Oakey, 1996). The bathymetry with its vicinity to the shelf break, the comparatively large water depth, the three-dimensional variability of stratification and tidal currents, and the interaction of the barotropic tide with bathymetry and stratification creates a complex environment that is not easily compared to partially mixed estuaries.

In their combination, the moored and shipborne current and stratification measurements of Geyer et al. (2000) and the turbulence observations of this paper provide approximately closed budgets of momentum, energy and salt. The circulation in the uniform stretch of the Hudson River which we probed during the experiment resembles the classical view of estuaries from the 1950s and 60s (for references see Section 1). This major result still leaves one important question unresolved: did we detect the bulk of mixing and exchange across the halocline, and are the observations of mixing representative of the whole
estuary? Alternatively, is mixing in the Hudson concentrated in local “hot spots”? Limited spatial surveys discussed herein do not hint at the dominance of hot spots. However, topographically-induced flow response with enhanced turbulence was clearly observed even if its role in longer-term larger-scale averages was probably insignificant. The five Hudson cruises mainly probed a minor bathymetric feature. Potentially more important with respect to mixing is the complex river bathymetry south of the George Washington Bridge, which we surveyed only once during ebb. Hydraulic processes in this area appear to be especially important during flood (Chant, 1995; Chant and Wilson, 1997). Variations of turbulence associated with these processes remain to be explored. In this context it would be especially interesting to investigate whether secondary transverse currents affect mixing in a similar, if likely less extreme, way than in the Tacoma Narrows of Puget Sound (Seim and Gregg, 1997).

The turbulence variables most relevant to the physical environment are the turbulent fluxes of momentum and mass (rather than, e.g., the dissipation rate). Being able to realistically estimate these fluxes from microstructure or other turbulence measurements is thus highly important. As shown above, estimating the momentum flux or turbulent stress from $\varepsilon$ is not a problem in low-Ri flows. However, the estimated flux of salt and similar scalars depends on, and is sensitive to, the way the flux Richardson number is modeled, $R_f = R_f(Ri)$ herein. Further laboratory, numerical and/or field experiments with direct flux measurements appear necessary to reduce the uncertainty in $R_f$. The traditional usage of constant $R_f$ is impermissible in estuaries. It can be shown that the salt flux values in P97, which are based on $R_f = \text{const.}$, are significantly overestimated in the lower third of the water column.

Even though inverse correlations between gradient Richardson numbers and mixing parameters appear in this paper, it is also shown that $Ri$ is generally a poor quantitative indicator of mixing and sometimes not even a qualitative indicator. The long streamwise survey discussed in Section 4 shows especially clearly that simple recipes of prescribing eddy coefficients in the form of $K_m = K_m(Ri)$ and $K_p = K_p(Ri)$, such as in the venerable study of Munk and Anderson (1948), are inadequate as parameterizations of turbulent mixing. Rather, a dynamical model of the turbulence is required. A current investigation is underway to determine how well a “$k - \varepsilon$” turbulence closure model adapted from Regener et al. (1997) and Burchard and Baumert (1995) is able to simulate mixing in the Hudson Estuary. This effort is based on the observations that microstructure measurements and turbulence closures largely use the same variables and similar simplifying assumptions, that verifications of the closure scheme need to be done against quantitative observations of turbulent mixing, and that model verifications based on observed stratification and currents alone are plainly insufficient.

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