Interactions and Growth of Normal Faults: Comparison of Model with Observations

Chrysanthe Spyropoulos,¹
Department of Applied Physics and Applied Mathematics, Columbia University, New York, NY, USA.

Isabelle Manighetti
Institut de Physique du Globe, Laboratoire de Tectonique et Mecanique de la Lithosphere, Paris, France.

Juan Contreras
Departamento de Geología, Centro de Investigación Científica y de Educación Superior de Ensenada, Ensenada, BC, México

Christopher H. Scholz and Bruce E. Shaw
Lamont Doherty Earth Observatory, Columbia University, New York, NY, USA.

Abstract. We have developed a numerical model of crack growth in a brittle layer extended over a ductile substrate which successfully reproduces the statistics of crack populations and their evolution with brittle strain. Here we study the crack interactions in the model and compare them with geological observations of systems of sub-parallel normal faults. Most of the geological examples are found in the model, but because we can observe their temporal (or strain) evolution in the model a deeper understanding can be obtained than that from geological observations or static crack models. We observe, in end pinning, a progressive increase in the crack tip tapers and slowing of crack propagation with progressive overlap. Cracks pinned at both ends become increasingly sessile and accrue strain by accumulating slip with little or no lengthening. Small offset cracks attract, then pin one another, eventually developing echelon segmented arrays with sessile cracks in the interior. Cracks become inactive when bypassed and stress shadowed by a neighbor. When cracks coalesce, they may develop saddles in their slip distributions which can be quite persistent throughout additional growth, as observed on real normal faults. Complex slip distributions may result from multi-fault interactions in regions of high brittle strain, such as in the Afar rift region, and the systematic asymmetry of the slip distributions there may be explained by a strain rate gradient.

Introduction

Faults seldom occur as isolated features, but within fault systems or populations. As a result, faults commonly interact with their neighbors through their stress fields, which results in complex behavior that cannot be treated by modeling the fault as the propagation of an isolated crack. A full understanding of fault system behavior requires these interactions to be taken into account [Jackson and McKenzie, 1983; Walsh and Watson, 1991; Armijo et al., 1996]. In particular, fault interactions play an essential role in forming fault controlled geologic structures, such as rift basins [Jackson and Leeder, 1994; Gupta et al., 1999; McLeod et al., 2002; Roberts and Yielding, 1994; Dawers and Underhill, 2000; Contreras et al., 2000] and in understanding seismic hazard [Harris and Day, 1993; Harris, 1998; Scholz and Gupta, 2000]. There have

¹Now at Exxon-Mobil Upstream Research, Houston TX, USA.
been many field studies of fault interactions. Many such studies have been of systems of subparallel normal faults, because good exposures of such systems are common [Peacock and Sanderson, 1994; Dawers and Andersen, 1995; Cartwright et al., 1995; Ackermann and Schlische, 1997; Crider and Pollard, 1998; Gupta and Scholz, 2000a] [Scholz, 2002, pp. 126-135]. The primary evidence for fault interaction obtained in these studies are distortions of the fault slip distributions from that expected for a model of an isolated crack, or, in the case of extreme interactions, in the presence of secondary faults thought to have so resulted. A fundamental drawback to such studies is that they are snapshots in time. Reflection suggests that stress interactions will affect not only the slip distributions but also the propagation velocities of the faults. Fault propagation will lead to a time (strain) dependent history of fault interactions, a non-trivial effect, particularly if more than two faults are interacting. Furthermore, fault propagation interactions will determine the positions of fault tips with respect to one another, and hence define the overall form of the fault system as it evolves through time. Fault interactions have been modelled with elastic crack models [Segall and Pollard, 1980; Bürgmann et al., 1994; Willemsse et al., 1996; Willemsse, 1997; Crider and Pollard, 1998]. However these models are also snapshots and hence tell only part of the story. They do not predict or include fault propagation or the effect of fault interactions on propagation velocity. This is an inherent weakness in elastic crack models. It results from the fact that they predict a stress singularity at the crack tip, a non-physical result, so they cannot be used to predict crack propagation. We have developed a numerical model of the extension of an elastic-brittle layer coupled to a ductile substrate in which cracks spontaneously nucleate, propagate, and accrue slip in response to an imposed uniaxial extension. Thus far we have explored only the statistical properties of the crack populations produced by this model [Spyropoulos et al., 2002], and how they evolve with strain. At the low brittle strains [Scholz and Cowie, 1990] the model develops a crack population characterized by a power law fault length distribution [Spyropoulos et al., 2002]. At higher strain, where fault interactions dominate over nucleation and growth, there is a gradual transition to a regime in which the distribution of the longest fault lengths is exponential. At very large strains, a saturation regime develops which is characterized by equally spaced system-size cracks. All three of these fault populations have been observed in nature. The transition from the power law to the exponential regime with increasing (brittle) strain has been duplicated in a physical model [Spyropoulos et al., 1999] and has also been observed in nature for populations of normal faults [Gupta and Scholz, 2000b]. The model, and its applicability to normal faults, has therefore been well validated. Here we examine the interior workings of the model and compare examples of fault interactions found there with the results of geological observations of potentially similar cases. Because in the model we can examine the temporal (strain) history of each case we are able to illuminate such interactions more fully than previously and to test previously proposed interaction mechanisms.

The Model

Our model is a two dimensional quasi-static system where cracks nucleate and propagate in response to deformation of the ductile layer. The displacement field is given by a scalar field in the direction of extension. This scalar displacement field models tension cracks. We argue that the model is applicable to normal fault interactions because the symmetry of the crack driving stress fields of tensile (mode I) cracks and normal faults (mode III shear cracks) are the same. In Figure 1 we sketch the main features of the fields of the driving stresses for both type of cracks: crack normal tension for tension cracks and anti-plane crack parallel shear for the case of normal faults. Our results, we claim, are sensitive only to the symmetry of these stress fields. Regions of positive stress drop will impede or repel other cracks and regions of negative stress drop will attract them, for both modes. We have already tested this by showing that the predictions of the model apply to normal fault populations. Here we will rely on the agreement of the model results to the observations to make this case anew. Figure 2 shows a schematic representation of the discretized model, described in detail in Spyropoulos et al. [2002]. It is a two dimensional model of spring blocks in which all lengths scale with W, the thickness of the brittle layer. The leaf springs coupling the blocks to the ductile layer approximate the fracture energy G of cracks growing in 3 dimensions. Cracks open a distance h when the local stress reaches a yield threshold given by \( \Phi_0(x, y) \), a random distribution of strengths. Once a crack opens, its yield strength is given by a slip weakening criterion defined by an inverse slip parameter \( \alpha \) which represents crack breakdown and friction:

\[
\Phi = \Phi_0 - \frac{\alpha h}{1 + \alpha h}, \quad h \geq 0. \tag{1}
\]

It is shown in Spyropoulos et al. [2002] that \( \alpha \) is the only parameter to which the model is sensitive. This parameter determines the strain localization. As \( \alpha \to 1 \) the cracks approach elastic cracks, having elliptical slip
distributions and stress tip singularities, and run away, resulting in a system of a few system-size cracks. As \( \alpha \to 0 \) there is no localization and one obtains only small cracks at the scale of the disorder in the yield strength. Our earlier work showed that between these extremes, stable, qualitatively similar results were obtained over a broad range of \( \alpha \). The results we present in this paper are not sensitive to the value of \( \alpha \) used within this broad range. In our earlier work, we showed that we could scale out the heterogeneity by considering the strain scaled by the heterogeneity [Spyropoulos et al., 2002]. Here, however, we just use unscaled strain, so its significance is as the relative value in a sequence of strain values.

Figure 3 shows the slip distribution on an isolated crack, growing in a homogeneous strength field along the plane of the crack. It is symmetrical, with slip tapering to the ends in a manner similar to that given by the Dugdale-Barenblatt model [Cowie and Scholz, 1992] and for much the same reason: the slip weakening criterion insures that the stress drop tapers to zero as the crack tip is approached. This slip distribution results in finite stresses at the crack tip and the crack propagates stably. The model slip distribution near the tip, where a reversed curvature appears, is not realistic and should be ignored. Slip distributions near the tip of normal faults, and in some cases along large portions of faults, have consistently been shown to be linear tapers [Dawers et al., 1993; Manighetti et al., 2001]. This “ogee” (reversed curvature) distribution, which occurs also in the Dugdale-Barenblatt model, results from the requirement of both of these 2D models of having to quash the singularity by a slip distribution in the fault plane. In reality, the singularity is quashed by inelastic deformation in a process zone volume surrounding the crack tip [Vermilye and Scholz, 1998], and models that permit such volume deformation predict, correctly, a linear taper to the crack tip [Scholz, 2002, pp. 16-17 and 121] Including in the model two effects left out of Figure 3, strength heterogeneities and fault interactions, gives a more realistic behavior, with more linear like slip profiles, as well as more realistic crack populations. Map views of typical crack populations at different strains are shown in Figure 4. In this and subsequent figures that show model results, all distances are scaled by the brittle layer thickness.

**Results**

Our model produces not only the statistical results published earlier [Spyropoulos et al., 2002] but the position and slip distribution of all faults within it. Our procedure, therefore, was to examine field studies of fault configurations and then to look into the model results to find examples which seemed to mimic, at least in their geometric relationships, those field examples. This turned out not to be difficult; a rather cursory search could lead to many such examples.

**End pinning**

Interactions between overlapping echelon normal faults are the most commonly observed and studied. The example shown in Figure 5, of a main bounding system of the Malawi Rift, is unique in that the slip distributions of the faults have been determined at three time intervals, separated by millions of years, by backstripping of the seismic stratigraphy of the basin [Contreras et al., 2000]. The south and north faults are offset 5 and 7 km, respectively, from the central fault, a fraction of the brittle crust thickness, which in this locality is 30 km as determined by seismicity [Jackson and Blenkinsop, 1997; Zhao et al., 1997]. The north and south faults appear to be single pinned, interacting at only one end. Their slip distributions become asymmetric, with steeper tapers at the interacting ends which progressively become steeper with increasing slip. This effect was first pointed out by Peacock and Sanderson [1994], has been studied systematically by Gupta and Scholz [2000b], and modelled by Willemse [1997]. The effect on fault propagation can be most clearly seen for the south fault. It propagates more slowly to the north, eventually becoming pinned. The central fault is pinned at both ends, propagating very little but continuing to produced brittle strain by increasing its D/L (displacement to length) ratio. This latter effect is thought to result in much of the scatter in D/L ratios in large data sets [Schlische et al., 1996; Willemse et al., 1996; Cartwright et al., 1995]. In Figure 6 the same phenomena are illustrated from model data. Slip (opening) profiles are shown for a number of cracks at four strain levels. The cracks (as in subsequent figures) are parallel and offset by a fraction of the width unity of the brittle layer. Three main cracks are shown in Figure 6, which form part of a larger area of interacting cracks. As in the Malawi Rift case in Figure 5, the two leftmost cracks are pinned in length, and grow mainly by increasing their slip and thus their D/L ratios. The rightmost crack continues to extend in length as well as in slip with increasing strain. There are three interesting aspects to its growth which are all similar to the Malawi case. First, there is an asymmetry in the slip profile, and the less steeply tapered side is the one that grows. Second, as it extends in length, it picks up even more slip than its pinned neighboring cracks. Third, the peak in slip shifts progressively to the right as the crack extends in that direction.
Segmentation and small faults

Figure 7 illustrates another feature which occurs where cracks overlap and interact. Here, we image the fault system in plane view with a grey scale proportional to the slip, for faults which have been recently active. This helps to visualize the smaller active faults, while neglecting relic old faults. In this figure, we see clearly that the small active faults are closely associated with the larger faults, and cluster, in particular, at stepovers and ends of ruptures. Harris and Anders [1995] observed many small faults near the interaction zones of major strands of offset normal faults, and suggested that these were produced by the stress concentration of the interacting tips. Our model results support their interpretation. The model thus evolves toward a segmented crack system, with small linking cracks insuring long term strain compatibility. The crack system grows primarily by the nucleation or attraction of new cracks at the ends of the system while the internal cracks become pinned at both ends and primarily develop by increasing their D/L ratio. We thus end up with a long en echelon segmented crack with slip profile similar to a single longer crack, just as observed by Contreras et al. [2000] and Dawers and Anders [1995]. These results demonstrate how such segmented crack systems develop; by a 'pile-up' of cracks in which the inner segments become sessile and new segments collect at the tips of the pile-up. Such segmented crack systems are commonly observed for both joints [Pollard et al., 1982; Vermilye and Scholz, 1995] and normal faults [Dawers and Anders, 1995; Manighetti et al., 1998; Contreras et al., 2000]

Stress shadowing

In the previous section, we saw that both the propagation and slip of a fault is inhibited when it enters the stress drop region of a nearby fault. What then happens to a fault that is entirely within the stress drop region of a neighbor? Ackermann and Schlische [1997] partially addressed this question. They showed cases in which the stress drop region of a fault is devoid of smaller faults, suggesting that nucleation cannot occur within the stress drop region. On the earthquake timescale, the issue addressed by Harris and Simpson [1996, 1998] is the same: no earthquakes will nucleate in the stress drop region of an adjacent one until tectonic stresses have erased the stress drop. These are fairly special cases, however. In the case demonstrated by Ackermann and Schlische [1997], the larger fault must have predated the generation of the smaller faults. In the Harris case, the stress drop is essentially instantaneous. We can examine this question more fully with model results. In Figure 8a we show the slip distributions at three different time (strain) steps of a number of cracks with spatial relations shown in Figure 8b. Two long cracks, b and c, are interacting. Fault b propagates past the smaller fault a. It shows its interaction with fault a by having a local slip minimum that is approximately equal to the slip of fault a. This appears to confirm a suggestion of Manighetti et al. [2001] that a slip deficit on one fault is taken up by slip on a neighbor. Fault a, meanwhile, remains static: it has become enveloped by the stress drop field of fault b and become inactive. Thus the stress shadowing effect is manifested on both faults and depends on both the relative size and position of each fault. Fault b is eventually pinned by fault c, a fault of similar length to fault b. In this case, we see that the small fault is dominated by the larger fault, even though the small fault pre-existed the larger one. This example lends a cautionary note to the example in the previous section: small faults in the overlapping region of larger faults may not have occurred as a result of that interaction, but may simply have been swept up into it. Thus field relations, the mainstay of geology, may sometimes be misleading.

Coalescence

When faults coalesce, either through 'hard linking' from secondary faulting, or through their stress fields, do they retain any 'memory' of this? Dawers and Anders [1995] first addressed this question. They pointed out that there should be a minimum of slip in the coalescing region, but, in the example they studied, did not find it. Manighetti et al. [2001], however, found many examples of minima within the slip distribution of faults, which they interpreted as revealing the coalescing regions between segments (or faults) that operated independently earlier in the evolution of the fault or system but later became linked. They also showed that such slip variations reflecting the earlier evolution of a fault can persist for at least a few million years as the fault is growing and interacting with others. An example is shown in Figure 9, while Figure 10 shows similar features in our model. The two faults in Fig. 10a are colinear (on the same grid line) and propagate towards one another, coalescing at the point indicated by the arrow at some time step between Figs. 10a and 10b. A saddle is observed in the slip distribution at the coalescence point, and, as can be seen in Figs. 10c and 10d, is maintained well after the coalesced fault continues to grow and interact with other faults. This example thus confirms that it is feasible to interpret the saddle in a slip distribution, such as shown in Fig. 9, as a relict of earlier fault coalescence at that point.
Complex slip distributions

In their study of normal faults in Afar, Manighetti et al. [2001] found a variety of complex slip distributions which they classified into a few simple forms. Most of these slip distributions are asymmetric, with long, linear portions. Manighetti et al. [2001] discussed various ways in which such asymmetric, quasi-linear profiles could form. Their favored scenario suggests that "barriers" may exist that have the ability of pinning the faults at various positions along their length, while, where such barriers do not exist, faults grow by developing a linear slip profile. On the other hand, the statistics of the faults in the Afar region was studied by Gupta and Scholz [2000b]. They found that the brittle strain in the central part of Afar exceeded 12%, and that above 8% the strain was produced by an increase in the D/L ratio of the faults without significant introduction of new faults. They also showed that faults in the low strain region have a power law size distribution, whereas in the high strain region (Asal rift) the faults have an exponential length distribution, as predicted by our model [Spyropoulos et al., 2002]. These later observations suggest a high degree of fault interactions in the Asal rift and that at high strains many faults must be pinned as in the case of the central fault in Fig.5. To investigate the possibility that some of the slip profiles observed by Manighetti et al. [2001] may result from fault interactions, we searched our model results for similar forms. These are shown, at two different time steps, in Figure 12. These results, producing similar forms as those in Fig.11, suggest that some of the observed slip patterns may indeed result from fault interactions, possibly by more than one fault. For example, in the trapezoidal form shown in Figure 12d, the fault to the left was offset farther from the central fault than the fault at the right. Hence the left interaction was weaker, the two fault tip slip tapers gentler, and the overlap is greater than for the corresponding interaction on the right. Similar interpretations can be made for the other examples shown in Figure 12. These results suggest that fault interactions can produce complex slip distributions similar to those observed. The ‘barriers’ in these cases are then the tips of neighboring interacting faults. Of course, other heterogeneities such as volcanic structures or cross faults can act as barriers, and have been shown to do so in the field [Manighetti et al., 2001].

Systematic asymmetry of the Afar region fault slip distributions

Manighetti et al. [2001] also observed a systematic asymmetry in the slip distributions in the Afar region. They found (Figure 13) that the point of peak slip for faults of length > 2km was systematically skewed towards their SE tips, but no such asymmetry was observed for shorter faults. Eastern Afar, where the measurements were done, is opening like a wedge and rifting is propagating to the NW [Manighetti et al., 1997, 1998]. Therefore there is a strain rate gradient, with the strain rate increasing to the SE. Manighetti et al. [2001] attributed the asymmetry in the peak slips to the strain rate gradient combined with propagation and ‘barrier’ effects. To investigate if such a strain rate gradient could result in this asymmetry in our model, we ran the model with strain rate gradient boundary conditions, as shown in Figure 14a. Map views of the cracks so generated are shown at two time steps in Figure 14b,c. In Figure 15 we show histograms of the position of the peak slip for (a) faults of length less than twice the brittle thickness and (b) faults longer than twice the brittle thickness (the choice of one or two brittle thicknesses as the cut-off did not significantly change the results). The results are qualitatively the same as the observations in Figure 13. Inspection of the model results showed that most of the faults propagated in a mainly unilateral manner in the direction down the strain-rate gradient. Hence they were slipping for a longer time, and slipped more, near their high strain rate ends. Because of the slip weakening feature of the model, this meant that their strength drops varied along length, being larger nearer their high strain rate ends. Thus the asymmetry in their slip distributions agree with the model results of Bürgmann et al. [1994] for the case of a stress drop gradient in the crack.

Discussion

We have shown that our model well produces and images interactions between sub-parallel cracks. The model slip profiles we obtain show many similarities with real slip profiles observed on active normal faults, suggesting that some of them may indeed result from fault interactions. The results of our model also agree qualitatively with the results of elastic crack models at distances far enough from the crack tip to avoid the problem of the stress singularity in the elastic crack model. Because cracks propagate in our model and we can look at the progression of interaction with time, we can gain additional insights into the crack interaction process which cannot be obtained with static stress modeling, and only rarely obtained by geological observations. The basic physics of crack interaction can be understood from their stress fields, shown schematically in Fig. 1. Consider subparallel cracks spaced close enough that as they approach their crack tips en-
ter the stress concentration (negative stress drop) region produced at the tip of the other. The propagation of those tips will then be enhanced, increasingly so as the tips approach. Thus two such cracks will attract one another. When each crack tip enters the stress-drop region of the other crack, its propagation will be inhibited. To propagate further, the stress at its proximal crack tip must now overcome both the yield strength of the material and the stress drop of its neighbor. In order to continue propagating, the crack tip taper must become steeper to generate this higher crack tip driving stress: hence such faults will develop the asymmetric slip distribution that is typical of this type of interaction. As the cracks propagate deeper into the stress drop regions of their neighbors, their proximal crack tip tapers become progressively steeper and their propagation rates progressively slower. Meanwhile, other cracks will be attracted to their distal tips, eventually overlapping and interacting repulsively. The inner cracks become increasingly sessile – increasing their D/L ratios with slower and slower propagation. This process continues, producing the echelon segmented crack systems typical of joints and normal faults. In the field, we seldom see cracks in the attractive interaction configuration because they will be in that position for a relatively short time and so are less likely to be preserved in the geological record. As such a system grows, smaller cracks that are further offset will become shadowed by the stress drop region and become inactive. Eventually, the only active cracks will be spaced several layer thicknesses apart, thus outside one another’s stress drop region, with small cracks in the vicinity of stepovers. By the definition used in our model interpretation, crack coalescence occurs only for cracks on the same or adjacent gridlines. After such a ‘true’ coalescence, the crack continues to grow as a single crack, but, as shown in Fig. 10, may for some time retain a distorted slip distribution with a saddle indicating the point of coalescence. Cracks that are offset farther, like those shown in Fig. 7, progressively acquire a collective slip distribution that begins to approach that of a single longer crack, as has been suggested by Dawers and Anders [1995] and Contreras et al. [2000]. Thus, they may be said to have coalesced through their stress interactions.

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Figure 1. Schematic map view of stress perturbations caused by crack, with stress increases at the ends and stress decreases off the sides.
Figure 2. Schematic of the model. Plan view: The blocks are connected with leaf springs in the $y$ direction and coil springs in the $x$ direction. The blocks can move in the $x$ direction only. A crack is shown as an opening of a certain distance $h_{ij}$ in the middle of block $(i,j)$. In cross section, there are leaf springs in the $z$ direction. The ratios of the spring coefficients, $k_x/k_z$ and $k_y/k_z$, set the resolution. The lower ductile layer is stretched and drives the upper brittle layer through the leaf springs.
Figure 3. Simulation of the growth of an isolated normal fault. $x$ axis is scaled by the brittle layer thickness, $W$. $y$ axis is scaled by the stress drop.
Figure 4. Plan view of a population of simulated normal faults at three different strains: (a) $\epsilon = 0.064$, (b) $\epsilon = 0.072$, (c) $\epsilon = 0.08$. The $x$ and $y$ axis are scaled by the brittle layer thickness, $W$. Line thickness is proportional to the amount of slip above a small threshold value.
Figure 5. (a) Three normal faults at Lake Malawi. The along-strike hangingwall displacement is shown at three time intervals. (b) Plan view of the configuration of the above faults as they appear at the last time interval [Contreras et al., 2000].
Figure 6. Evolution of simulation faults with a plan view configuration similar to that of the Malawi faults in Figure 5. Four different strains are shown: (a) $\epsilon = 0.004$, (b) $\epsilon = 0.006$, (c) $\epsilon = 0.008$, and (d) $\epsilon = 0.01$. (e) Plan view of the configuration of the above faults as they appear at the last time interval. In all the panels, only recently active faults are shown.
Figure 7. Plan view of faults. Greyscale is proportional to slip. Only recently active faults are shown.
Figure 8. Fault slip evolution showing stress shadowing. (a) Slip on faults at three time intervals superposed. (b) Plan view of configuration of faults.
Figure 9. Example of normal fault from Asal with a distinct saddle point in the slip displacement profile believed to be the result of coalescence of two faults.
Figure 10. Simulation of two faults coalescing. Arrow indicates coalescence location. Note persistence of slip deficit there.
Figure 11. Classification of various forms of slip distributions and their relative frequency for faults in the Afar region. Modified from similar data used in Manighetti et al. [2001].
Figure 12. Examples of various forms of slip distributions seen in the model. Compare with Figure 11.
Figure 13. Histogram showing the position of the maximum slip displacement for the faults in the Afar region study. The x axis is scaled by the length of the faults. (a) Faults with length < 2 km, (b) Faults with length > 2 km [Manighetti et al., 2001].
Figure 14. Evolution of model faults with gradient in the strain rate across the fault system. (a) Sketch of loading geometry. (b) and (c) map view of crack generated at two time steps.
Figure 15. Histogram of position of peak slip. The x axis is scaled by the length of the faults. (a) Fault lengths < 2W. (b) Fault lengths > 2W. Note similarities with Figure 13.