Implementing the Weak Temperature Gradient Approximation with Full Vertical Structure

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ABSTRACT

A two-column, nonrotating radiative–convective model is formulated in which the free-tropospheric temperature profiles of the two columns are assumed to be identical and steady and the temperature equation is used diagnostically to calculate the vertical velocities [the weak temperature gradient (WTG) approximation]. These vertical velocities and the continuity equation are then used to calculate the horizontal velocities. No horizontal momentum equation is used. This model differs from other two-column models that have used similar formulations in that here both columns are governed by the same laws rather than different dynamical roles being assigned a priori to the ‘‘warm’’ and ‘‘cold’’ columns. The current formulation has the advantage of generalizing trivially to an arbitrary number of columns, a necessity for developing a 3D model under WTG. The two-column solutions compare reasonably well with a reference two-column model that uses a linear, nonrotating horizontal momentum equation and the same underlying radiative–convective code as the WTG model; the reference model is essentially that used earlier by Nilsson and Emanuel, except modified to have significant viscosity only in a boundary layer near the surface. The two solutions compare best in the limit of large horizontal domain size, behavior opposite to what has been found in models that lack an explicit boundary layer and have viscosity throughout the troposphere. The difference is explained in terms of the circulation driven by boundary layer pressure gradients.

1. Introduction

Since horizontal temperature gradients are small in the Tropics, it seems reasonable to take them to be zero as a simplifying assumption in theoretical or idealized modeling studies. This has been done in a number of ‘‘two column’’ models that represent the Tropics by two homogeneous regions, representing the ascending and descending branches of the Hadley or Walker circulations (Pierrehumbert 1995; Miller 1997; Larson et al. 1999; Clement and Seager 1999; Kelly and Randall 2001). In more general terms, scaling arguments based on the horizontal uniformity of tropical temperature can be used to justify a variety of balanced dynamical models as approximations to the primitive equations (Charney 1963; Held and Hoskins 1985; Browning et al. 2000; Sobel et al. 2001; Majda and Klein 2003). We refer to approximations of this type collectively as the weak temperature gradient (WTG) approximation. We are aware of no existing three-dimensional numerical solutions of any WTG system but would like to obtain such solutions in the future.

Those studies that have used WTG in two-column models have used a formulation in which the two columns are assigned, a priori, different roles in the dynamics. The column with the greater sea surface temperature (SST) controls the free-tropospheric temperature in both columns through convective adjustment. The column with the lower sea surface temperature is then stable to deep convection and controls the circulation by the requirement that subsidence balance radiative cooling in that column. The ascent in the warm

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column must be of equal magnitude to the subsidence in the cold column, and the horizontal circulation between the two columns is also determined by mass conservation. We will call this the ''different jobs'' formulation. Although it captures the gross dynamical roles of the subsiding and ascending branches of the tropical circulation, it does not generalize to an arbitrary number of columns. However, there is nothing about the WTG approximation per se that should restrict it to two columns, and so we would like to have an algorithm that can be applied to an arbitrary number of columns as a step toward developing WTG models in three dimensions. We describe and show results from such an algorithm here. We also compare the WTG solutions with solutions from a model that has otherwise identical physics and numerics but solves a linear momentum equation for the circulation (with nonzero temperature gradient). Except for a change in the vertical structure of the viscosity coefficient (to be described below), this model is essentially identical to that of Nilsson and Emanuel (1999, hereinafter NE99). This model will be referred to as the ''reference model.'' Although scaling arguments give us good guidance as to when the WTG approximation should be accurate, it seems a good idea to check using explicit solutions with reasonable physical parameterizations, and we are not aware of another study that has done so in the two-column context despite the fact that several such studies have used WTG.

Section 2 contains a full description of the WTG model. We give a brief description of the reference model in section 3, discuss the results in section 4, and conclude in section 5.

2. Model description

a. Underlying model

We extended the WTG single-column radiative–convective model of Sobel and Bretherton (2000, hereinafter SB00), which is based on the single-column model of Rennó et al. (1994a,b), into a two-column model. The underlying temperature and moisture equations are, in pressure coordinates,

$$\frac{\partial T}{\partial t} + \mathbf{u}_h \cdot \boldsymbol{\nabla} T + \omega S = Q_T \quad \text{and} \tag{1}$$

$$\frac{\partial q}{\partial t} + \mathbf{u}_h \cdot \boldsymbol{\nabla} q + \omega \frac{\partial q}{\partial p} = Q_q, \tag{2}$$

where T is temperature, q is specific humidity, p is pressure, $\mathbf{u}_h$ is horizontal velocity, ∇ is the horizontal gradient operator, ω is vertical velocity, $S = (T/\theta)(\partial\theta/\partial p)$ is the static stability, θ is potential temperature, and Q_T and Q_q are the temperature and moisture forcings, respectively.

The single-column model uses the radiation parameterization of Chou et al. (1991) and Chou (1992) and the convective scheme of Emanuel (1991) to determine the forcings Q_T and Q_q. All parameters in these schemes are the same in the two columns. The radiative scheme assumes clear skies, a major limitation for realistic simulations but less important for the purpose of developing and testing the WTG algorithm to be described below, which is our primary purpose. Including a more sophisticated radiative scheme with an explicit cloud model, as has been developed by Bony and Emanuel (2001), would be straightforward. The time stepping is leapfrog with a Robert filter. A vertical resolution of 50 hPa was used in this study. More details of the original single-column model are described by Rennó et al. (1994a,b).

The model domain consists of two columns with equal areas, each of which is assumed to be horizontally uniform. The horizontal gradient [which will be neglected in (1) but retained in (2)] will be represented by two-point finite differences [see (8) and accompanying discussion below].

b. WTG model

The essential aspect of the WTG approximation is the well-known dominant tropical balance, above the planetary boundary layer (PBL), between mean vertical advection and heating, so that (1) becomes

$$\omega S = Q_T, \tag{3}$$

so that what was a prognostic equation for T becomes a diagnostic one for ω. However, (1) does not guarantee conservation of mass, because, although we can assume that the free-tropospheric S is equal in the two columns (since, at a given pressure, it depends only on the temperature profile, which is assumed to be horizontally uniform), at any given time step Q_T will, in general, not be equal and opposite in the two columns. Rather, there will be some horizontal mean heating, which should go into changing the horizontal mean temperature rather than into driving the circulation. Therefore we define a mean heating, $Q_{TM} = (Q_{T1} + Q_{T2})/2$, where the subscripts 1 and 2 refer to the two columns, and write

$$C\frac{\partial T_M}{\partial t} = Q_{TM} \quad \text{and} \tag{4}$$

$$\omega_i S_M = Q_{Ti} - Q_{TM}, \tag{5}$$

where $i = 1$ or 2. In the free troposphere, by assumption $T_M = T_1 = T_2$ and $S_M = S_1 = S_2$. The tendency of the mean temperature $\partial T_M/\partial t$ will not be assumed to vanish in general (although we explicitly describe only steady-state solutions, we arrive at these by time-dependent integration).

An extra fictional heat capacity C is added for numerical stability. The feedbacks among radiation, convection, and temperature are not as direct in this two-column system as in a single column or as in a two-column system that does not use WTG. In our two-column WTG system, the effects of any heating occurring in one column are ''diluted'' by being immediately distributed between both boxes, whereas without WTG, the temperature in each column responds directly only to heating in that column, though dynamical adjustment eventually will communicate the effects to the other column. WTG assumes this adjustment to be instantaneous, whereas in a non-WTG model it will occur in finite time. Since feedbacks between temperature and heating are generally negative [e.g., convection increases temperature, stabilizing the sounding to further convection; for more detailed discussion of convection–temperature feedbacks, see, e.g., Mapes (1997)]. By weakening such feedbacks, WTG destabilizes the system; increasing C is a convenient way of compensating for this. In many cases $C = 1$ (no additional heat capacity) is adequate for stable integration, but in some of our simulations values as large as 10–50 are necessary to stabilize the model during initial stages of the simulation when high-frequency transients occur in response to initial conditions that are far from the eventual steady state. In the steady state, the tendency term vanishes and the value of C has no relevance. At present, our algorithm is valid only to obtain steady solutions. With many columns, and careful initialization, we cautiously expect that the net heating Q_{TM} will be generally small for realistic boundary conditions and that stable integration can occur with $C = 1$, allowing fully time-dependent solutions in general. We postpone careful study of this to future work and limit ourselves to steady solutions here. We have verified by experimentation that the steady states described here are unique, independent of initial conditions, and insensitive to the precise value of C and the value of the time step.

We use (5) to compute ω above the PBL in each column. Here as in SB00, the PBL top is assumed to have a fixed pressure, chosen to be 850 hPa. In the PBL we let the temperature be different in the two columns and use (1) in its standard form, which means that ω must be determined by some means other than (5). As in SB00, the vertical velocity in the boundary layer is found by linearly interpolating ω between its value at 850 hPa, computed using (5), and zero at the surface.

The horizontal velocity is computed using the vertical velocities computed as described above in the continuity equation, which in this model is

$$\frac{\partial u}{\partial x} + \frac{\partial \omega}{\partial p} = 0, \qquad (6)$$

with u being the horizontal velocity and x being the horizontal direction. As in NE99, by the two-point differencing, our model has only one horizontal velocity, defined at the interface between the two columns (equivalent to assuming the existence of walls at the outside edges of each column, as opposed to periodic boundary conditions), so that

$$\frac{\partial u}{\partial x} = \frac{u}{L}, \qquad (7)$$

where L is the horizontal extent of each column. We use simple centered differences to approximate the vertical derivatives of ω. An interesting consequence of computing the horizontal velocity this way is that the horizontal advection term in the moisture equation, (2), is independent of the width of the columns. The advection term is approximated by

$$u\frac{\partial q}{\partial x} = \frac{u}{L}\Delta q = -\frac{\partial \omega}{\partial p}\Delta q, \qquad (8)$$

where Δq is the difference in specific humidity at a given level between the two columns. From the rightmost expression in the triple equality (8), it is easily seen that the horizontal moisture advection term is independent of L. As all other terms determining T and q are also independent of L, the total solution for T and q is as well, although u is not (Bretherton and Sobel 2002, hereinafter BS02). This is a direct consequence of using WTG instead of a momentum equation.

c. Different-jobs model

In the model described above, there is symmetry in the dynamics of the two columns in that they differ only in the boundary conditions and other imposed forcings. We also constructed a version of the model that implements the different-jobs formulation described briefly in the introduction. In this model, WTG is still used in that, above the PBL, the temperature is the same in the two columns. However, the column with lower SST determines the vertical velocity in the free troposphere using (3)—the mean is not subtracted in this case—while the other determines the free-tropospheric temperature using (1) and a vertical velocity equal in magnitude and opposite in sign at every level to that computed by the cold column. Again, the vertical velocity is linearly interpolated from the top of the boundary layer to zero at the surface, and the temperature is interactive (and different in the two columns) in the boundary layer. This procedure is similar to those used in previous WTG two-column studies (Pierrehumbert 1995; Miller 1997; Larson et al. 1999; Clement and Seager 1999; Kelly and Randall 2001).

d. Reference model

As in NE99, the reference model also uses the single-column code of Rennó et al. (1994a,b) for the underlying physics, but unlike our WTG model the reference model has prognostic temperature, allowing different free-tropospheric temperature profiles in the two columns, and uses a momentum equation to compute the velocities where, in the WTG model, we use only the WTG temperature equation, (3), and mass continuity, (6). To be specific, the reference model solves the linear, hydrostatic, and nonrotating equations of motion:

$$\frac{\partial u}{\partial t} = -\frac{\partial \Phi}{\partial x} + \frac{\partial}{\partial p}\left(\mu \frac{\partial u}{\partial p}\right), \tag{9}$$

$$\frac{\partial \Phi}{\partial p} = -\rho^{-1}, \quad \text{and} \tag{10}$$

$$\frac{\partial u}{\partial x} + \frac{\partial \omega}{\partial p} = 0, \tag{11}$$

where μ is the vertical viscosity, Φ is the geopotential height, and ρ is the density. The reader is asked to consult NE99 for more details.

Our model, using WTG, is implicitly nearly inviscid in the free troposphere [see Sobel et al. (2001) and BS02 for relevant discussions]. The temperatures in the PBL are allowed to differ in the two columns, implying a horizontal pressure gradient that may be significant there even if it is zero above the PBL. This must be balanced in steady state by viscosity [a crude model of turbulent vertical momentum fluxes; (9)] in the PBL. This balance is not explicitly present in the construction of the WTG model, since that model contains no explicit momentum equation. To obtain the most appropriate reference model with which to compare our WTG model, we altered the NE99 model to incorporate a vertically variable viscosity, with μ being nonzero only in the PBL. We set μ at the surface to correspond to a damping time scale of 1 day (see NE99) and linearly interpolate vertically to zero at 800 hPa (the first level above the PBL) and above. This change in the vertical structure of μ is the only significant difference between our reference model and that used in NE99.

Unlike in the WTG model, the horizontal advection term in (2)—and thus the total solution—in the reference model depends on L, which we took (except in sensitivity studies) to be 180° of longitude. This is appropriate for comparison with the WTG model (which, by assuming that pressure gradients are negligible while velocities are finite, essentially assumes that the free troposphere is inviscid) because, as NE99 showed, increasing the column width has the same effect as decreasing the viscocity, and WTG should be a good approximation in the quasi-inviscid regime.

3. Results

All three models—the reference model, the WTG model, and the different jobs model—were run with fixed SSTs varying separately in each column from 25° to 30°C in increments of 1°C. The carbon dioxide concentration was set to 330 ppm, the surface albedo was set to 0.05, the surface wind speed was set to 7 m s^{-1}, and the insolation was set to its annual average at 7° latitude.

a. Omega profiles

Figure 1 shows the ω profile of the descending column (the ascending column has equal and opposite omega) for a simulation with SSTs of 26° and 29°C. The WTG, reference, and different jobs models are shown. The agreement among all of them is fairly good. When the two columns have equal SST (not shown), both the WTG and reference models yield a radiative–convective equilibrium with precisely $\omega = 0$ at all levels in both columns, as they should. In some cases, the different-jobs model yields a small but nonzero circulation for equal SST in the two columns. This result is understandable since, despite the symmetry in the boundary conditions, there is an asymmetry in the model dynamics in this formulation. When the SST is uniform, the jobs are assigned arbitrarily, since there is no rational basis for assigning them. It is apparent that this arbitrary choice can have consequences for the steady solution.

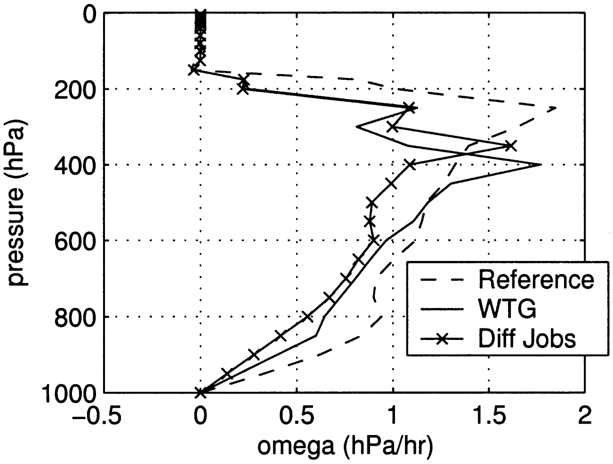

FIG. 1. Vertical velocity profiles; SST = 26° and 29°C.

b. Precipitation and temperature fields

Precipitation is closely related to the ω field, stronger upward motion being closely associated with more precipitation. Figure 2 shows curves of precipitation of one column with fixed SST while the SST of the second column is varied for the WTG model, and Fig. 3 shows an analogous set of curves for 500-hPa temperature. Each curve represents a different SST of the column with fixed SST. (There is redundant information in Figs. 3 and 5. The value at the point 25°C on the 27°C curve, e.g., is the same as the value at the point 27°C on the 25°C curve, as it should be since the difference corresponds only to a relabeling of the columns. In Figs. 2 and 4, these corresponding points again show the same simulation but are not redundant, since they show precipitation from different columns.) Figures 4 and 5 show the same curves for the reference model. For the precipitation, both the shapes of the curves and the actual values are very close when we compare the WTG and reference models. The temperature field is not as similar between the two models as the precipitation field is, though they still have a roughly similar shape and the

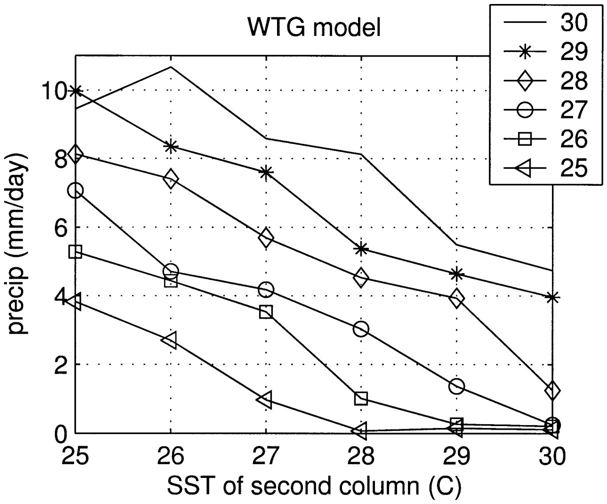

FIG. 2. Precipitation of one column as a function of the SST of the second column, for the WTG model.

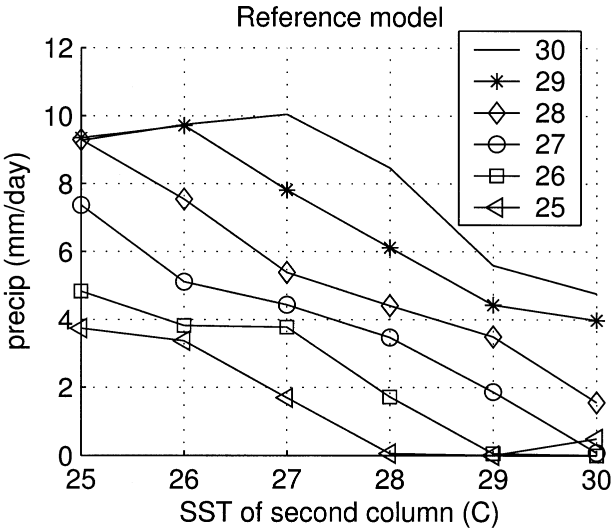

FIG. 4. Precipitation of one column as a function of the SST of the second column, for the reference model.

same range of temperatures. Much of the difference can be viewed as a simple offset between the two models, that is, the curve with the fixed SST of 30°C in the reference model is very similar to that with the fixed SST of 29°C in the WTG model, and so on. We do not currently understand the cause of this offset.

c. Dependence on domain size

Since the WTG model is independent of *L* but the reference model is not, we test how well the two models agree for smaller domain size. Figure 6 shows the temperature fields of the reference model with *L* set to 60° and 18° of longitude, which correspond to domains that are 3 and 10 times as small, respectively, as that used to obtain the results shown in the previous figures. The WTG model's fields obviously are the same as in Fig. 4. Comparison of the figures shows a slightly worse agreement between the two models with *L* at 60° than at 180°. When *L* is decreased to 18°, the agreement becomes still worse.

It is interesting that the WTG model is a better approximation to the reference model with (horizontally) bigger columns than with smaller columns. The model of BS02 is similar to that used here in that it represents a nonrotating Walker-type circulation under WTG, but it has a fixed vertical structure and a continuous horizontal dimension, whereas this study has many vertical degrees of freedom and just two horizontal points. BS02 found that WTG is a better approximation with smaller horizontal domain, while we find here that WTG is a better approximation with larger horizontal domain.

We reconcile this apparent contradiction by considering the different vertical profiles of the viscosity coefficients in the two studies. BS02 assume a vertically

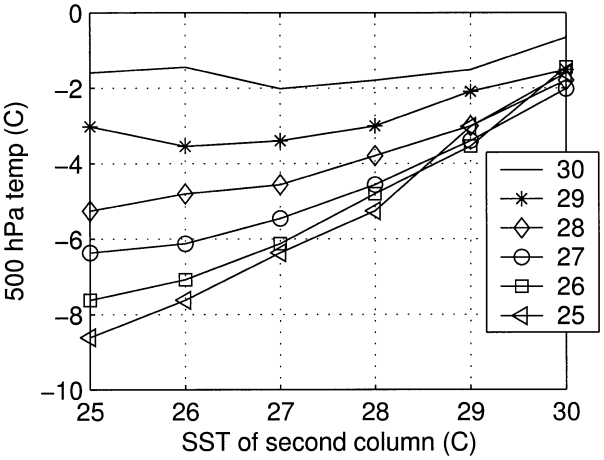

FIG. 3. The 500-hPa temperature of the two columns as a function of the SST of the second column, for the WTG model.

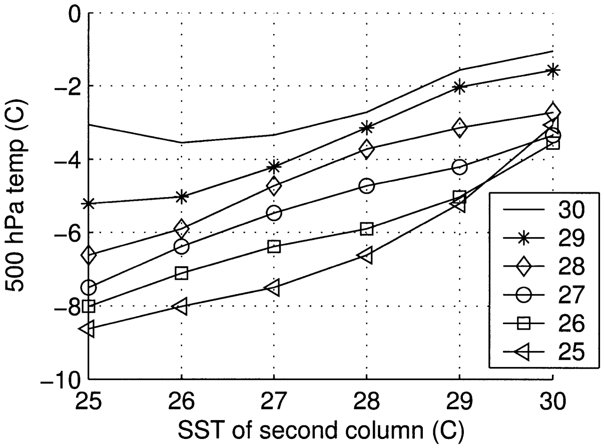

FIG. 5. The 500-hPa temperature of the two columns as a function of the SST of the second column, for the reference model.

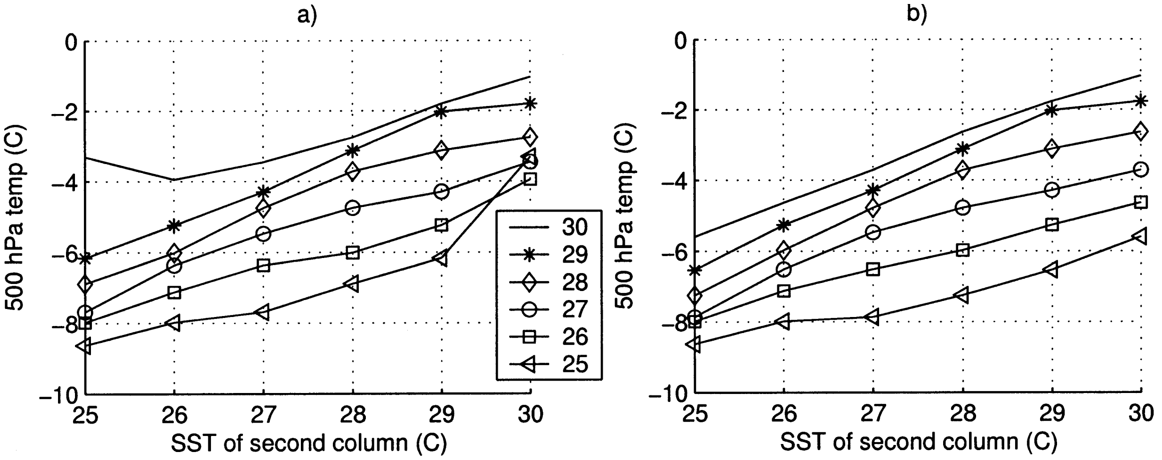

FIG. 6. The 500 hPa temperature of one column as a function of the SST of the second column. Reference model with L = (a) 60° and (b) 18° of longitude.

uniform viscosity and no explicit PBL. As the domain size increases, for fixed SST contrast, the tropospheric temperature *gradient* (which is determined implicitly under WTG) becomes smaller, but not as rapidly as the SST gradient does (since viscous drag is finite throughout the troposphere and balances the pressure gradient), so that at sufficiently large domain size the tropospheric temperature gradient is comparable to the SST gradient. This renders WTG an inappropriate approximation, since it is inappropriate to drive the circulation by the SST gradient while neglecting a free-tropospheric temperature gradient of equal magnitude. Here, the viscosity vanishes in the free troposphere, and so a negligible free-tropospheric temperature gradient is consistent even at very large domain size. At the same time, we include an explicit PBL in which deviations from WTG are allowed. In the reference model, for small domain size a circulation can be driven directly by the hydrostatic pressure gradients associated with the SST contrast. This effect, which we call the ‘‘Lindzen–Nigam effect’’ (Lindzen and Nigam 1987) requires an explicit momentum equation and so is not captured by the WTG model, which explains the poor agreement at small domain size. At large domain size, the hydrostatic PBL pressure gradient is reduced (for fixed SST contrast between the two columns) and so the WTG and reference models agree better in that regime.

The fact that agreement between the WTG and reference models is best at large domain sizes is not a major drawback in the sense that it would necessarily ruin simulations using a large number of columns. What is important is not the size of each column, but the size of the whole domain relative to the total SST contrast across the domain. For a 3D simulation on a domain similar to the Tropics of the earth, as horizontal resolution increases, column sizes will become smaller, but the SST contrast from one column to the next will also. As long as the total domain size remains fixed at a

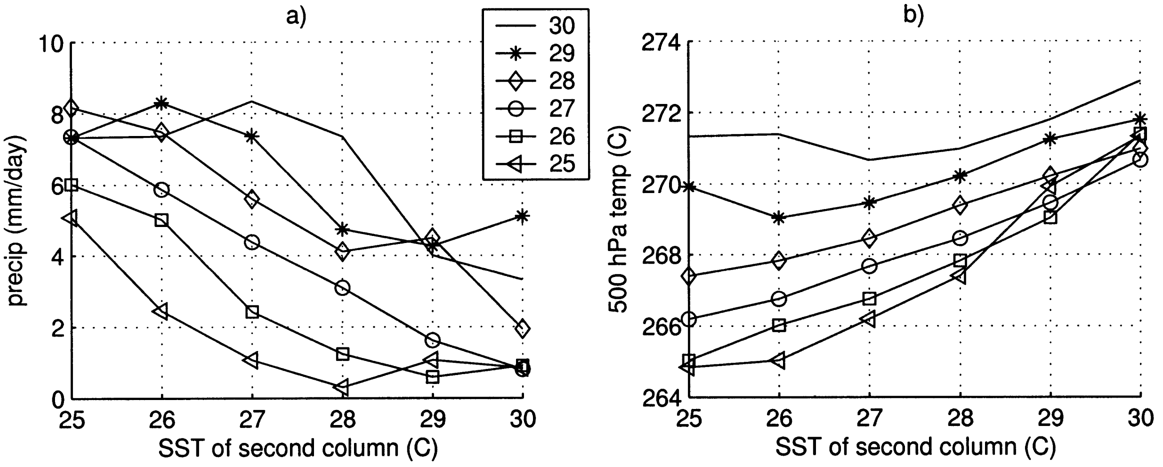

FIG. 7. (a) Precipitation and (b) 500-hPa temperature of one column as a function of the SST of the second column, for the different-jobs version of WTG model.

sufficiently large value, a good simulation may still be possible. At the same time, the inability of the WTG model to represent the Lindzen–Nigam effect will cause errors, particularly in surface winds. It may be desirable to develop a scheme for explicitly including PBL momentum dynamics while retaining WTG in the free troposphere.

d. Different-jobs version

In all of the fields described above, the regular WTG model compares somewhat better to the reference model than the different-jobs version does. Figure 7 is an example of this result, showing the precipitation and temperature fields of the different-jobs version of the WTG model. Comparing Figs. 3 and 4 with Fig. 7 shows that the precipitation field of the standard WTG model has better agreement with the reference model than does the different-jobs version of the WTG model. The temperature field has only slightly better agreement. Nonetheless, these results indicate that our basic WTG model captures the ‘‘full’’ dynamics of the reference model better than does the different-jobs version, at least for this set of simulations. This outcome is consistent with our notion that the basic WTG model is more general since it allows the radiative and convective processes to play the same roles in the dynamics wherever they occur, as presumably they do in nature where the laws of physics are invariant with respect to position.

4. Conclusions

We constructed a two-column model that uses the WTG approximation and thus does not require a horizontal momentum equation. The model dynamics are formulated identically in the two columns, so that, unlike the previously used different-jobs implementation of WTG in this context, the approach generalizes to an arbitrary number of columns, as will be necessary for full three-dimensional simulations. We compared our model with a version of NE99’s model, referred to above as the reference model, which used a linear momentum equation to determine the horizontal velocity but otherwise has identical physics and numerics to ours, under fixed sea surface temperature. For large horizontal domain size, the two models agree reasonably well, though quantitative differences exist. In particular in the precipitation field, our approach agrees with the reference model better than the different-jobs model does.

Our WTG solutions agree best with the reference solutions in the limit of large horizontal domain size. This result is in contrast to solutions with a single vertical degree of freedom and continuous horizontal structure (BS02) in which WTG was found to be a consistent approximation only for small domain size. The difference appears to be due to the confinement of viscosity to the PBL in the reference solution here. The absence of any viscosity in the free troposphere prevents the breakdown of WTG there at large horizontal scales (although we include in our WTG solutions a PBL where WTG is not enforced). The lack of an explicit momentum equation in the PBL instead causes the WTG model to agree poorly with the reference solutions at small horizontal scales.

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