Department of Applied Physics and Applied Mathematics
Columbia University
APPH E4210. Geophysical Fluid Dynamics
Spring 2005

Problem Set 1

(Due Feb 3, 2005)

1. *Dispersion relation*. Find the dispersion relation for the following equations (subscripts represent partial derivative, and ∇^2 is the 3-dimensional Laplacian):

   (a) $\psi_{tt} = c^2\nabla^2\psi + \alpha^2\nabla^2\psi_{tt}$. (Longitudinal waves in bars.)

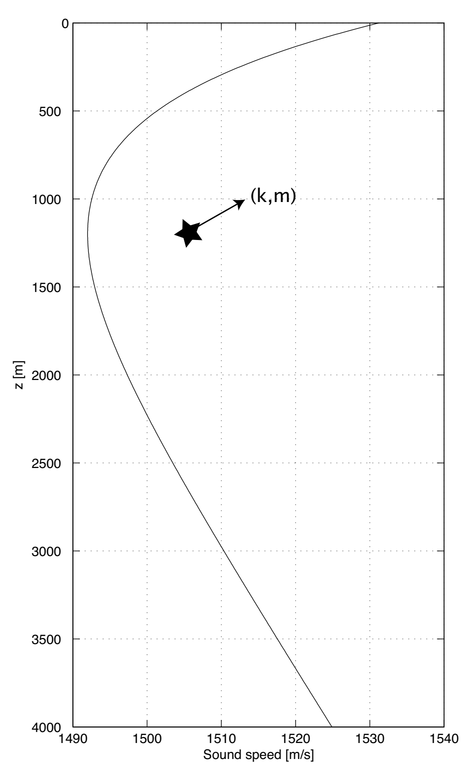
   (b) $\partial(\nabla^2\psi - \psi/R^2)/\partial t + \beta\psi_x = 0$. (Rossby waves.)

   (c) $\psi_{tt} = c^2(\psi_{xx} - \alpha\psi_{xxxx})$. (Piano string.)

2. *Ray tracing*. (This is not a very GFDish example, but its the simplest one I could think of for illustrating ray tracing.) Consider the equation for sound waves in the ocean:

$$\psi_{tt} = c^2(\psi_{xx} + \psi_{zz})$$

Here, x points in the horizontal direction, and z in the vertical. We will assume that the sound speed, c, only varies in the vertical direction. A typical sound speed profile is shown in the figure below.



(a) For a "slowly-varying" wavepacket, find the dispersion relation.

(b) Write down the ray-tracing equations for this problem, i.e., the equations that allow you to compute how frequency (ω) and wavenumber ($\mathbf{k} = (k, m)$) change following the path of the wavepacket (i.e., a "ray").

(c) Using the fact that the sound speed is only a function of z, make a *sketch* of a ray originating at the location marked by the star symbol in the above figure, and directed as shown by the arrow in the figure. *Hint*: Rearrange the dispersion relation to express m as a function of ω and k. Note that sound waves are transverse waves, that is the direction of energy propagation (the ray direction) is parallel to the wavenumber vector.

(d) For extra credit, write a small computer program to numerically integrate the ray equations. (Matlab is very convenient for this.) To do this you will need to know $c(z)$. The following Matlab code fragment generates the sound speed profile shown in the figure.

```
% vertical grid
dz=5;
z=[0:dz:4000]'; % depth in meter
% Set up sound speed profile. This particular one
% is the Munk profile but any will work.
B=1200; z0=1200.0;
c0=1492.0;
ep=0.006;
eta=2*(z-z0)/B;
c=c0*(1+ep*(eta+exp(-eta) -1)); % sound speed
```

For definiteness, assume that the source (located at 1200 m) generates sound waves at a frequency $\omega_0 = 47.5$ radian/sec, and a horizontal wavelength $2\pi/k$ of 200 m.