

Department of Applied Physics and Applied Mathematics
Columbia University
APPH E4210. Geophysical Fluid Dynamics
Spring 2005

Problem Set 7

(Due April 14, 2005)

1. Consider the statement for conservation of potential vorticity (PV) for a shallow layer of fluid (see figure):

$$\frac{D}{Dt} \left(\frac{\xi + f}{H} \right) = 0.$$

- (a) Show that in the limit of small Rossby number, this statement is equivalent to the quasigeostrophic potential vorticity equation (QGPVE):

$$\frac{\partial q}{\partial t} + J(\psi, q) = 0,$$

where, the potential vorticity q is given by

$$q = \nabla^2 \psi - \frac{1}{\lambda_d^2} \psi + \beta y + f_o \frac{h_B}{H_o},$$

and, $\psi = g\eta/f_o$ is the streamfunction. (The fact that the O(1) flow is nondivergent allows us to define a streamfunction in this manner.)

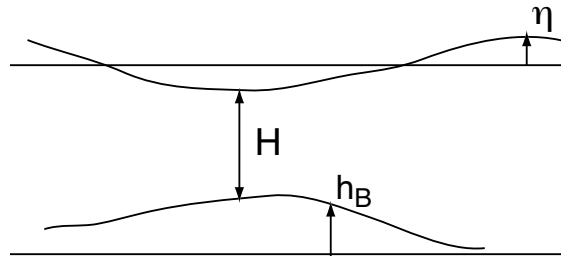


Figure 1: Definition sketch for problem 1.

2. Show that a single plane Rossby wave is an exact solution of the nonlinear QGPVE (see problem 1). What about a *sum* of plane waves?
3. Write down the linearized version of the QGPVE. Assuming that the ambient potential vorticity, $Q = \beta y + f_o h_B/H_o$ has a constant *gradient*, find the dispersion relation for quasigeostrophic Rossby waves.