Department of Applied Physics and Applied Mathematics
Columbia University

APPH E4210. Geophysical Fluid Dynamics

Spring 2005

Problem Set 9

(Due May 5, 2005)

1. In this problem you will complete the solution of the Eady baroclinic instability problem. To refresh your memory: we are considering quasigeostrophic motions of a uniformly rotating (f-plane), uniformly stratified (N constant) fluid bounded by two horizontal surfaces at $z = 0$ and $z = H$. The instability problem considers small perturbations about a background state consisting of a steady, uniformly sheared, zonal flow $\overline{u}(z) = (U_o/H)z$. This flow is in thermal wind balance with a horizontal density (temperature) gradient.

   (a) Make a contour plot of the growth rate ($\mathrm{Im}\,\omega$) as a function of k and l. (Nondimensionalize appropriately.) Note that the fastest growth occurs when $l = 0$. Wave motion is then purely in the meridional direction (a consequence of the O(1) horizontal velocity field being nondivergent), i.e., down the mean temperature gradient, and the release of available potential energy is maximized.

   (b) In the Eady problem, long waves, i.e., waves with μH less than a critical value, are unstable (c is complex).

      i. Find the full solution $\psi'(x, y, z, t) = \mathrm{Re}\,\hat{\psi}(z)\exp i(kx + ly - \omega t)$ for the unstable waves. (It is convenient to write $\hat{\psi}(z) = |\hat{\psi}(z)|\exp i\alpha(z)$.)

      ii. For the most unstable wave, make a vector plot (in the y-z plane) of the velocity field (v',w'). (You will need to pick a particular value of x.)

   (c) For a disturbance with $k = l$ (a so called "square Eady wave"), find the maximum growth rate and wavelength of the most unstable perturbation. Assuming a buoyancy frequency of $N = 10^{-2}\mathrm{s}^{-1}$, what is the e-folding time (in days) for growth for this wave?