Department of Applied Physics and Applied Mathematics
Columbia University
APPH E4210. Geophysical Fluid Dynamics
Spring 2008

Problem Set 6

(Due Apr 10, 2008)

1. In this problem you will complete the solution of the Eady baroclinic instability problem. Recall that we are considering quasigeostrophic motions of a uniformly rotating (f-plane), uniformly stratified (N constant) fluid bounded by two horizontal surfaces at $z = 0$ and $z = H$. The instability problem considers small perturbations about a background (basic) state consisting of a steady, uniformly sheared, zonal flow $\overline{u}(z) = (U_{\mathrm{o}}/H)z$. This flow is in thermal wind balance with the basic state density (temperature) structure.

   (a) Make a contour plot of the growth rate ($\mathrm{Im}\,\omega$) as a function of k and l. (Nondimensionalize the growth rate by the advective timescale in the problem, $\lambda_{\mathrm{d}}/U_{\mathrm{o}}$.) Note that the fastest growth occurs when $l = 0$. Wave motion is then purely in the meridional direction (a consequence of the O(1) horizontal velocity field being nondivergent), i.e., down the mean temperature gradient, and the release of available potential energy is maximized.

   (b) In the Eady problem, long waves, i.e., waves with μH less than a critical value, are unstable (c is complex).

      i. Find the full solution $\psi'(x, y, z, t) = \mathrm{Re}\,\hat{\psi}(z) \exp i(kx + ly - \omega t)$ for the unstable waves. It is convenient to write $\hat{\psi}(z) = |\hat{\psi}(z)| \exp i\alpha(z)$.

      ii. For μH corresponding to the most unstable wave, make plots of $|\hat{\psi}(z)|$ and $\alpha(z)$ for both the growing and decaying solutions. Indicate in which direction the phase lines tilt with height for the two solutions. Note that $|\hat{\psi}(z)|$ takes on a minimum value at $z_c = H/2$. At this height, known as the *steering level*, the phase speed is equal to the local mean flow, i.e., $c_r = \overline{u}(z_c)$.

      iii. Calculate the meridional heat flux, $\rho_{\mathrm{o}} C_p \overline{v'\theta'}$, associated with the disturbance. (C_p is the specific heat capacity at constant pressure of air.) Show that a wave with phase lines tilting westward with height is associated with poleward heat flux, that is, a growing disturbance transports heat poleward (as it must if it is to draw upon the potential energy stored in the mean flow).

      iv. Using values of various parameters typical of the mid-latitude atmosphere, compute the poleward heat flux associated with the fastest growing wave.

v. For the most unstable wave, make contour plots (in the x-z plane) of the pressure field (ψ') and the temperature perturbation (θ'). Note the characteristic tilt of the phase lines. Also note that at the surface ($z = 0$), there is a phase shift between the temperature and pressure perturbations associated with the disturbance, with warm air just ahead (westward) of the pressure trough.

vi. For the most unstable wave, make a vector plot (in the y-z plane) of the velocity field (v',w'). (You will need to pick a particular value of x.)

(c) For a disturbance with $k = l$ (a so called "square Eady wave"), find the maximum growth rate and wavelength of the most unstable perturbation. Assuming a buoyancy frequency of $N = 10^{-2}\text{s}^{-1}$, what is the e-folding time (in days) for growth for this wave?